

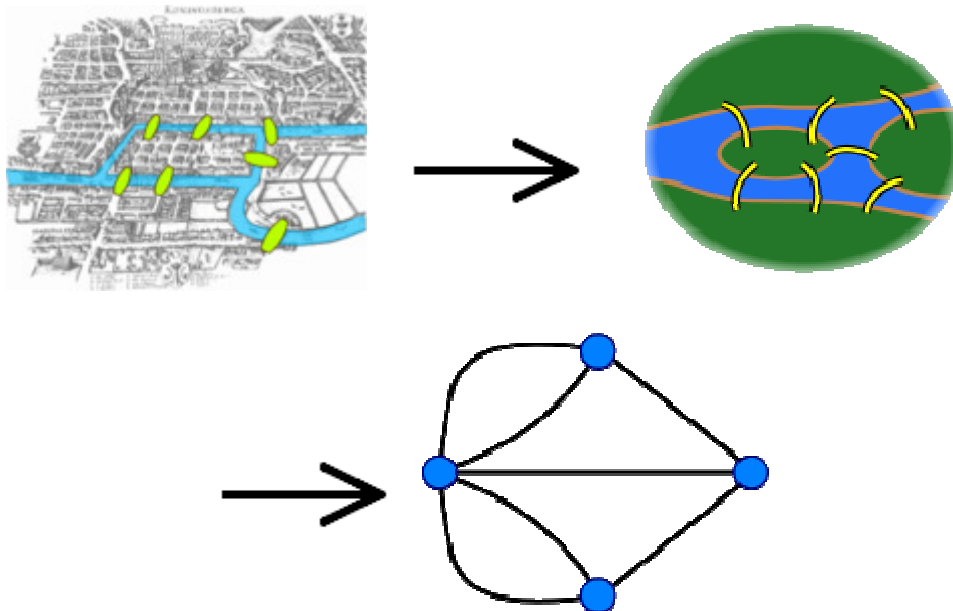
# The Königsberg Bridge Network

In the 18<sup>th</sup> century, the East Prussian city of Königsberg (now known as Kaliningrad, on the Baltic Sea in a small sliver of Russian territory sandwiched between Poland and Lithuania) had seven bridges across the Pregel (or Pregolya) River, which runs through the city.



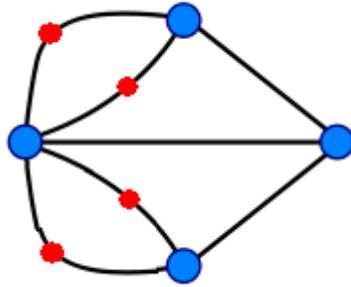
(Source: [http://news.bbc.co.uk/2/hi/europe/country\\_profiles/6177003.stm](http://news.bbc.co.uk/2/hi/europe/country_profiles/6177003.stm))

The original question was to find a path through the city which crosses over each of these seven bridges exactly once, and Euler reduced the problem to a question about an edge – vertex graph; specifically, one first eliminates all features but the land masses and the bridges connecting them, and then one represents each land mass with a vertex and each bridge with an edge whose endpoints are the two land masses it connects.



(Source: [http://en.wikipedia.org/wiki/Seven\\_Bridges\\_of\\_K%C3%B6nigsberg](http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg))

The final object in this sequence can be converted into a graph by adding four vertices as indicated in the drawing on the next page:



In Unit **IV** we prove that there is no path of the desired type by translating everything into the language of algebraic chain groups for graphs; the argument can be viewed as a formalization of the original impossibility proof due to Euler in **1736**.