Applications to graphs

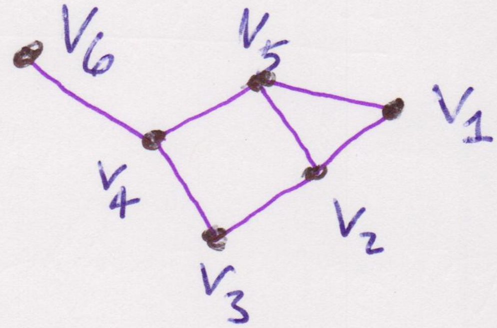
(These are **not** graphs as defined and studied in coordinate geometry or calculus, but the name has stuck and is almost certain to remain unchanged.)

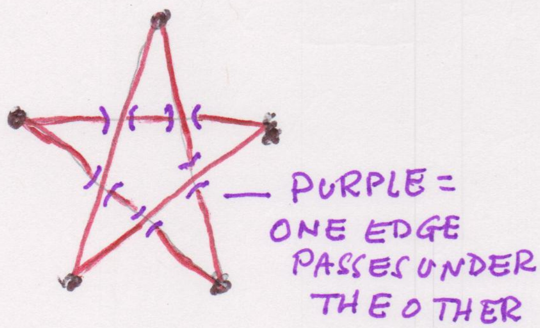
A (finite) edge-path **graph** is a compact T_2 space X with a finite family of closed subsets $\mathcal{E}$ (the <u>edges</u>) such that

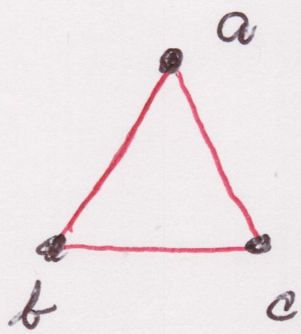
1. Each $E \in \mathcal{E}$ is homeomorphic to $[0,1]$
2. If $E_1 \neq E_2$ in $\mathcal{E}$, then $E_1 \cap E_2 = \begin{cases} \emptyset \text{ or a} \\ \text{common} \\ \text{endpoint} \end{cases}$
3. $$X = \bigcup_{E \in \mathcal{E}} E$$

The endpoints of the E's are called <u>vertices</u>.

EXAMPLES.



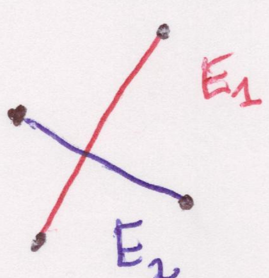




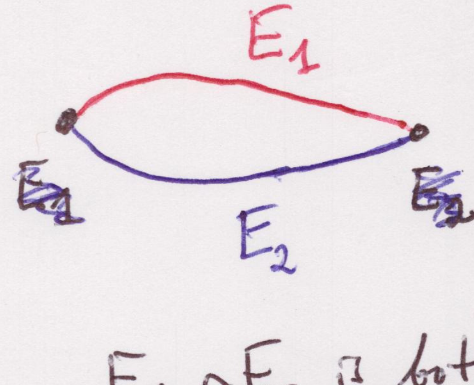
Complete graph on 5 vertices. For each pair of vertices there is an edge joining them.

(Later in the course we shall prove that the complete graph on 5 vertices is not homeo to a subset of the plane!)

NONEXAMPLES



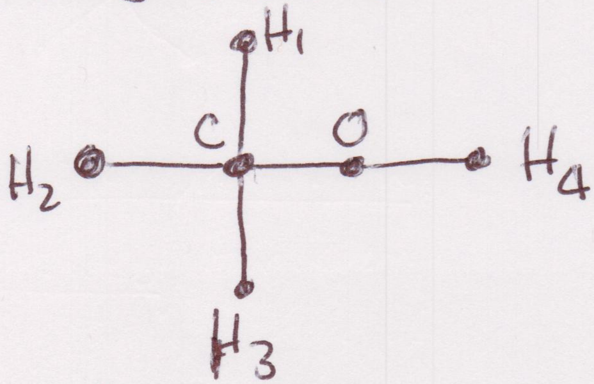
$E_1 \cap E_2$ is not an endpoint of either



$E_1 \cap E_2$ is both vertices of each edge.

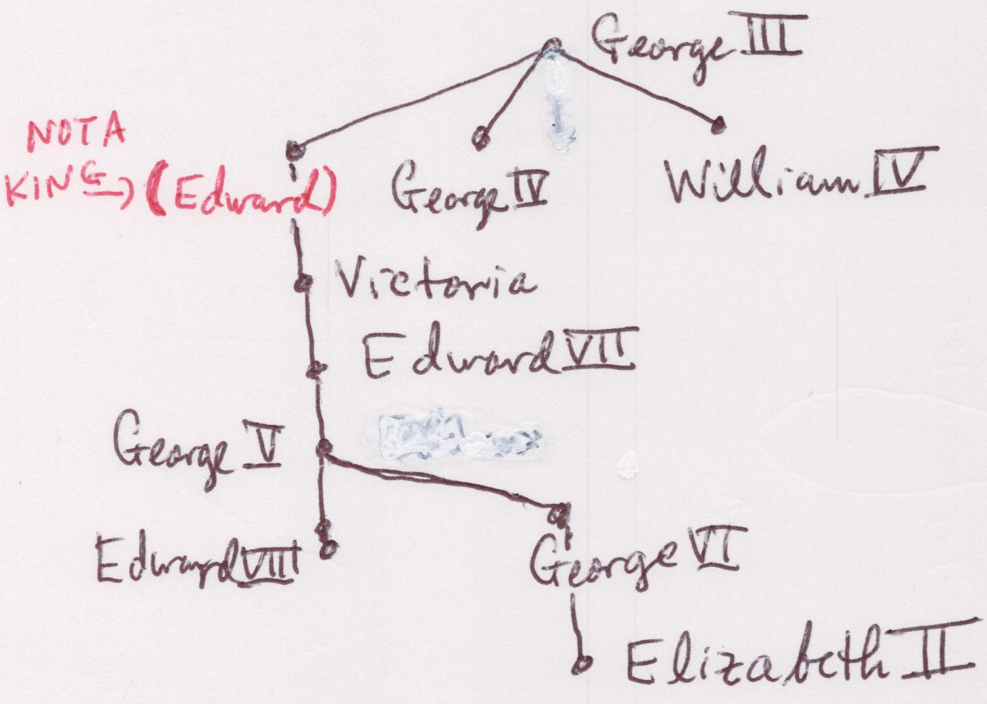
Such objects (a.k.a. <u>linear graphs</u>) are used in many subjects to model a range of phenomena. Oversimplified examples:

<u>Chemistry and physics</u>



Methyl alcohol molecule

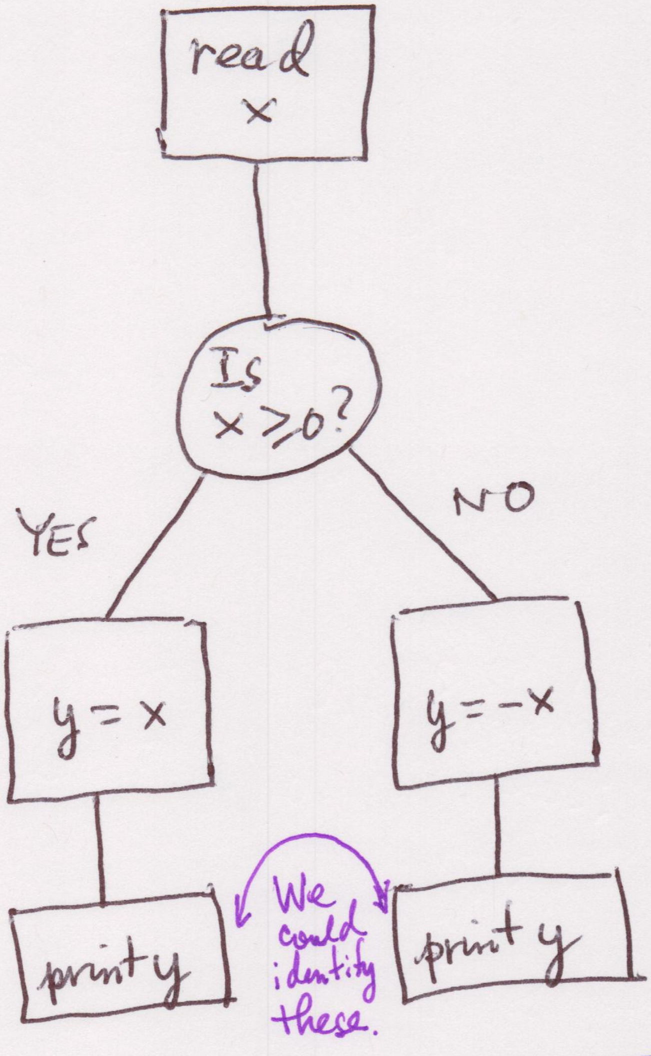
<u>Social sciences & humanities</u>



Kings and Queens of England genealogy chart

Computer science

Flow chart for computing $|x|$:



Obviously, the real-life applications of graphs will be significantly more complex.

The roles of graphs in this course:

1. Illustrate the uses of fundamental groups to analyze spaces.
2. Anticipate the development of homology theory in the rest of this course.

(Transitional topic)

<u>CLAIM:</u> Every graph is metrizable.

<u>IDEA:</u> Suppose $(X, \mathcal{E})$ has n vertices $v_1, \ldots, v_n$. Define $X' \subseteq \mathbb{R}^n$ such that if $E \in \mathcal{E}$ has vertices v_i and v_j, then X' contains the closed segment E' joining the standard basis vectors e_i and e_j. Check that this defines a cont 1-1 map $X \to X'$.

The discussion is continued in

graphs.pdf

(course directory).