

# Elementary expansions of graphs

Given a finite graph  $X$  and a family of edges  $F$ , one can construct a 2 – dimensional elementary expansion complex  $\mathbf{Expand}(X, F)$  by adjoining 2 – simplices along the edges of  $F$ . One can then prove directly that the homology groups of the graph  $X$  and the 2 – dimensional complex  $\mathbf{Expand}(X, F)$  are isomorphic. Furthermore, since each edge of a 2 – simplex  $A$  is a strong deformation retract of  $A$ , it follows that  $X$  is a strong deformation retract of  $\mathbf{Expand}(X, F)$ ; the retraction and the homotopy deforming  $\mathbf{Expand}(X, F)$  to  $X$  can be constructed over each 2 – simplex individually. In the example illustrated below, the graph  $X$  is the black triangle and the 2 – simplices in  $\mathbf{Expand}(X, F)$  which correspond to the edges of  $X$  are shaded in pink.

