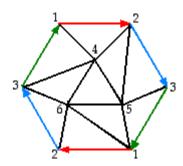
Triangulating the real projective plane



http://www.math.ust.hk/~mamyan/graphic/323 002.gif

In this discussion we shall view the real projective plane as the quotient space of the 2 – disk by the equivalence relation which identifies diametrically opposite points on the boundary. The drawing above represents a minimal triangulation of the real projective plane; the colored arrows represent pairs of edges that are identified under the equivalence relation.

If we rename the vertices 1, 2, 3, 4, 5, 6 as A, B, C, D, E, F respectively, then twice the 1 – cycle AB + BC - AC is the boundary of the following 2 – chain in the standard chain complex for the given triangulation of the real projective plane with the alphabetical ordering of its vertices:

However, there is no 2 – chain whose boundary equals AB + BC - AC. It is possible to prove this directly from the definition of the boundary map d_2 from 2 – chains to 1 – chains, but later in the course we shall give a more conceptual and less computational proof in the exercises.