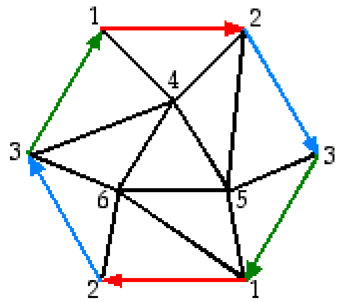
# Triangulating the real projective plane




http://www.math.ust.hk/~mamyan/graphic/323_002.gif

In this discussion we shall view the real projective plane as the quotient space of the $2$ – disk by the equivalence relation which identifies diametrically opposite points on the boundary. The drawing above represents a minimal triangulation of the real projective plane; the colored arrows represent pairs of edges that are identified under the equivalence relation.

If we rename the vertices $1, 2, 3, 4, 5, 6$ as **A, B, C, D, E, F** respectively, then twice the $1$ – cycle **AB + BC – AC** is the boundary of the following $2$ – chain in the standard chain complex for the given triangulation of the real projective plane with the alphabetical ordering of its vertices:

**ABD + BCD – ACE + ABF + BCF – ACD + DEF –BDE – AEF + CDF**

However, there is no $2$ – chain whose boundary equals **AB + BC – AC.** It is possible to prove this directly from the definition of the boundary map $d_2$ from $2$ – chains to $1$ – chains, but later in the course we shall give a more conceptual and less computational proof in the exercises.