# Elementary expansions of graphs

Given a finite graph **X** and a family of edges **F,** one can construct a **2 –** dimensional elementary expansion complex **Expand(X, F)** by adjoining **2 –** simplices along the edges of **F.** One can then prove directly that the homology groups of the graph **X** and the **2 –** dimensional complex **Expand(X, F)** are isomorphic**.** Furthermore**,** since each edge of a **2 –** simplex **A** is a strong deformation retract of **A,** it follows that **X** is a strong deformation retract of **Expand(X, F);** the retraction and the homotopy deforming **Expand(X, F)** to **X** can be constructed over each **2 –** simplex individually**.** In the example illustrated below**,** the graph **X** is the black triangle and the **2 –** simplices in **Expand(X, F)** which correspond to the edges of **X** are shaded in pink**.**
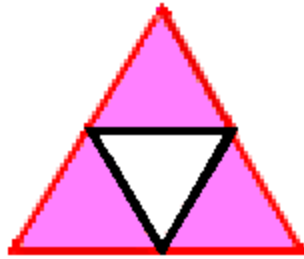
Exercise. Show that the inclusion of the triangle in the expanded complex induces an isomorphism in simplicial homology.

Hint: Show that if we add the 2 - simplices one at a time, then each successive inclusion has this property; this can be done using Mayer-Vietoris sequences.