Complements of standardly embedded subspheres

$$S^k \subseteq \mathbb{R}^{k+1} = \mathbb{R}^{k+1}\times\{0\} \subseteq \mathbb{R}^{k+1}\times\mathbb{R}^{n-k} \cong \mathbb{R}^{n+1}$$

$$S^k \xrightarrow{\subseteq} S^n \to \mathbb{R}^{n+1}$$

__Problem__ Show $S^n - S^k \cong S^{n-k-1}\times\mathbb{R}^{k+1}$

__Solution__ Write a point in $\mathbb{R}^{n+1}$ as (x, y) where $x \in \mathbb{R}^{k+1}$ and $y \in \mathbb{R}^{n-k}$

It suffices to show that $S^n - S^k \cong S^{n-k-1}\times\mathring{D}^{k+1}$, where $\mathring{D}^{k+1}$ is the open disk of radius 1 centered at the origin of $\mathbb{R}^{k+1}$ (since $\mathring{D}^{k+1} \cong \mathbb{R}^{k+1}$).

Let $f : S^{n-k-1}\times\mathring{D}^{k+1} \longrightarrow S^n$ be given by $f(u, v) = \left(v, \frac{1}{\sqrt{1-|v|^2}}\cdot u\right)$. Then f is 1-1 with image $S^n - S^k$. The inverse map $S^n - S^k \longrightarrow S^{n-k-1}\times\mathring{D}^{k+1}$ sends (x, y) to $\left(\frac{y}{|y|}, x\right)$.

$\boxed{\text{Note: } S^n - S^k = \text{all } (x,y) \text{ s.t. } |x|^2+|y|^2 = 1 \text{ and } y \neq 0.}$ ■