page 91, lines 10-11

Proof of Proposition 3.

NOTATION. If $e_0, \ldots, e_r$ are the vertices of Δ_r, then $\mathrm{Front}_p(\Delta_r) =$ simplex $e_0 \ldots e_p$ and $\mathrm{Back}_q(\Delta_r) =$ simplex $e_{r-q} \ldots e_q$ (OK for all $r \geq p, q$).

(i) Naturality w.r.t. cont maps $h : (X,A) \to (Y,B)$.

Let $h : (X,A) \to (Y,B)$ be cont, let $f \in S^p(Y,B)$ and $g \in S^q(Y,B)$, and let $T : \Delta_{p+q} \to X$. Then

$$[h^\#(f \cup g)](T) = f \cup g(h \circ T) \text{ by def.},$$

and the latter is $f(\mathrm{Front}_p(hT)) \cdot g(\mathrm{Back}_q(hT)) =$
$h^\# f(\mathrm{Front}_p(T)) \cdot h^\# g(\mathrm{Back}_q(T)) = [h^\# f \cup h^\# g](T)$.

Hence the functions $h^\#(f \cup g)$ and $h^\# f \cup h^\# g$ are equal (since we showed it for free generators).

This cochain lies in $S^{p+q}(X,A)$ because it vanishes if $T:\Delta_{p+q}\to A$.

(ii) <u>Bilinearity.</u> $[(f_1+f_2)\cup g](T) =$ $\boxed{\begin{gathered}T:\Delta_{p+q}\to X\\ \text{as before}\end{gathered}}$

$[f_1+f_2](\mathrm{Front}_p(T))\cdot g(\mathrm{Back}_q(T)) =$

$f_1(\mathrm{Front}_q(T))\cdot g(\mathrm{Back}_q(T)) + f_2(\mathrm{Front}_q(T))\cdot g(\mathrm{Back}_q(T))$

$= [(f_1\cup g) + (f_2\cup g)](T)$, so the cochains are equal. Similarly to prove $f\cup(g_1+g_2) = (f\cup g_1)+(f\cup g_2)$.

(iii) <u>Associativity</u>. T as above $\Longrightarrow$ $[(f\cup g)\cup h](T)$ and $[f\cup(g\cup h)](T)$ both equal $f(T|e_0\ldots e_p)\cdot g(T|e_p\ldots e_{p+q})\cdot h(T|e_{p+q}\ldots e_{p+q+r})$.

(iv) <u>Augmentation is a unit</u> $T:\Delta_p\to X \Longrightarrow$

$$\left.\begin{aligned} f\cup\varepsilon(T) &= f(T)\cdot\underbrace{\varepsilon(T|\{e_p\})}_{1} = f(T)\\ \varepsilon\cup f(T) &= \underbrace{\varepsilon(T|\{e_0\})}_{1}\cdot f(T) = f(T)\end{aligned}\right\}\text{all } T.$$