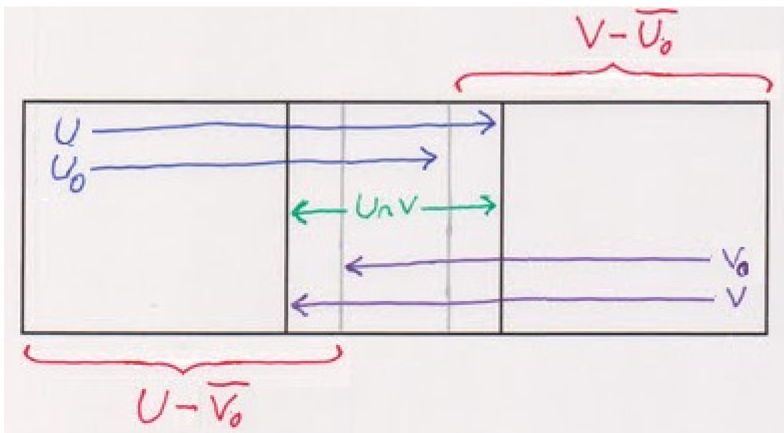
Drawing for the Mayer – Vietoris property in de Rham cohomology

The property in question is established on page 137 of the main course notes:

http://math.ucr.edu/~res/math246A-2012/advancednotes2012.pdf

Let **U** and **V** denote open subsets in $\mathbb{R}^n$. The key insight in the proof of the Mayer – Vietoris Property is to use a smooth partition of unity $\{g_U, g_V\}$ for the standard open covering **{U, V}** of the open set $U \cup V$. In the drawing below, U_0 and V_0 are the sets on which the functions g_U and g_V are positive.



Given a differential form ω, the form $\omega_V = g_U \cdot \omega$ can be extended to all of **V** by setting it equal to zero on **V – Closure(U₀)** because ω_U is zero by construction on the overlap

$$U \cap (V - \textbf{Closure}(U_0))$$

and similarly the form $\omega_U = g_V \cdot \omega$ can be extended to all of **U** by setting it equal to zero on the complement $U - \textbf{Closure}(V_0)$ because ω_V is zero by construction on the overlap

$$(U - \textbf{Closure}(V_0)) \cap V.$$

It follows that the differential form

$$(\omega_V | U \cap V) + (\omega_U | U \cap V) = g_U \cdot \omega + g_V \cdot \omega = (g_U + g_V) \cdot \omega$$

is merely the original form ω (since $\{g_U, g_V\}$ is a partition of unity).■