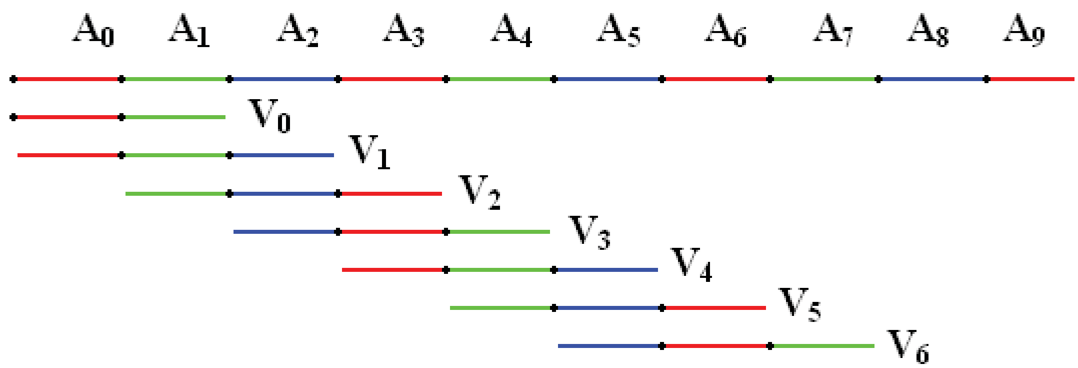
V : Cohomology and Differential Forms

V . 4 : De Rham's Theorem

The following drawing is designed to illustrate the decomposition of an open set $\mathbf{U}$ in Theorem **2** of the notes. We are given a presentation of the open set as a union of compact subsets $\mathbf{K_i}$, each of which is contained in the interior of the next one in the sequence. The sets $\mathbf{A_i}$, which correspond to the colored bands, are obtained by removing the interior of $\mathbf{K_{i-1}}$ from $\mathbf{K_i}$



The sets $\mathbf{V_i}$ are open neighborhoods of the sets $\mathbf{A_i}$, and they are constructed so that $\mathbf{V_i}$ and $\mathbf{V_j}$ have points in common only if $|\mathbf{i}-\mathbf{j}|$ is at most **2.** In the proof of Theorem **2** one uses these sets to find smaller open neighborhoods $\mathbf{W_i}$ of $\mathbf{A_i}$ such that $\mathbf{W_i}$ is contained in $\mathbf{V_i}$ and $\mathbf{W_i}$ is a finite union of open disks. Thus the families

$$\{W_0, W_3, W_6, \dots\} \quad \{W_1, W_4, W_7, \dots\} \quad \{W_2, W_5, W_8, \dots\}$$

consist of pairwise disjoint open subsets, each of which is a finite union of convex open subsets. We know that de Rham's Theorem holds for each $\mathbf{W_i}$ by previous results in the notes, and Proposition **3** implies that the theorem also holds for the unions $\mathbf{G_0}$, $\mathbf{G_1}$, $\mathbf{G_2}$ of the open sets in each of the three families. Theorem **4** gives the final steps to showing that de Rham's Theorem is true for finite unions of these sets $\mathbf{G_i}$ and hence is true for the arbitrary open subset $\mathbf{U}$ that we are considering.