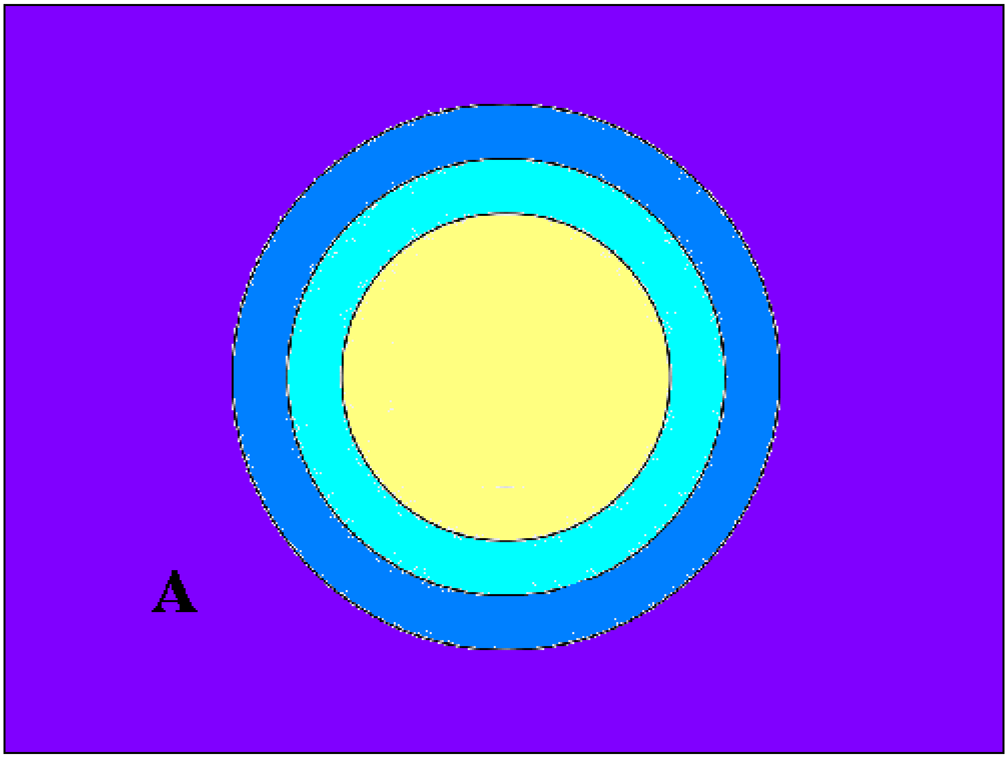
Cell attachment and homology

This drawing illustrates the sets in the argument on page 70 of the notes.



The entire rectangle is **X,** while **A** is the purple shaded region and the cell $\mathbf{e^k}$ is given by the interiors of the three concentric circles. The open set **V(3/4)** corresponds to the union of the purple set **A** and the dark blue annulus (without the frontier at which it meets the light blue annulus), while the closure of the set **V(1/2)** is the union of these with the light blue annulus. Note that **A** is a deformation retract of both **V(1/2)** and its closure, and the open set **V(3/4)** lies in **V(1/2)** (which is contained in the interior of its closure).

One then has that the inclusion map induces isomorphisms in homology from **(X, A)** to **(X, B = Closure V(1/2)),** and excision induces isomorphisms in homology from the latter to **(X – V(3/4), B – V(3/4)).** On the other hand, the latter pair is homeomorphic to the pair **(D(3/4), A(1/2, 3/4)),** where **D(r)** is a disk of radius **r** and **A(p,q)** is the annulus of all points whose distance from the center is between **p** and **q,** and the latter pair is homotopy equivalent to the pair $\mathbf{(D^k, S^{k-1})}$.