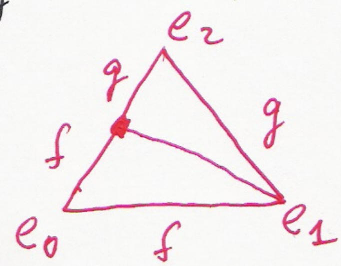
Additivity of the Hurewicz map on $\pi_1(X,x)$

The discussion on page 166 of Hatcher gives a good conceptual description of the proof. For the sake of completeness we shall describe things more explicitly.



Drawing on p.166 of Hatcher.

The line joining e_1 to the midpoint of segment $[e_0e_2]$ consists of all points whose barycentric coords. satisfy $t_0 = t_2$. Perpendicular projection onto the line e_0e_2 sends the point $t_0e_0 + t_1e_1 + t_2e_2$ to

$$(t_0 + \tfrac{1}{2}t_1)e_0 + (t_2 + \tfrac{1}{2}t_1)e_2 = P(t_0, t_1, t_2)$$

Hence if we start with closed curves $f, g : [0,1] \to X$, the singular 2-simplex T with faces $\partial_0 = g$, $\partial_1 = f+g$, $\partial_2 = f$ is given by $T(t_0e_0 + t_1e_1 + t_2e_2) =$

$$f+g\Big(P(t_0, t_1, t_2)\Big).$$