# FIGURES FOR ALGEBRAIC TOPOLOGY LECTURE NOTES

## I : Foundatonal and geometric background

### I.2 : Barycentric coordinates and polyhedra

***Barycentric coordinates.*** In the drawing below, each of the points **P, Q, R** lies in the plane determined by $\mathbf{P_1}$, $\mathbf{P_2}$, and $\mathbf{P_3}$, and consequently each can be written as a linear combination $w_1\mathbf{P_1} + w_2\mathbf{P_2} + w_3\mathbf{P_3}$, where $w_1 + w_2 + w_3 = 1$. In the picture below, for the point **P** the barycentric coordinates $w_i$ are positive, while for the point **R** the barycentric coordinates are such that $w_1 = 0$ but the other two are positive, and for the point **Q** the barycentric coordinates are such that $w_1$ is negative but the other two are positive.

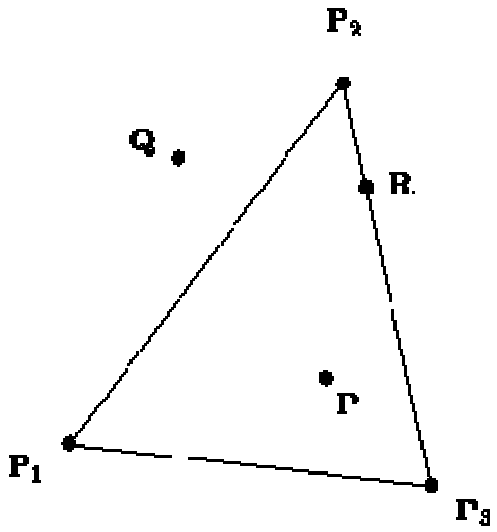



**FIGURE 1**

(***Source:*** http://graphics.idav.ucdavis.edu/education/GraphicsNotes/Barycentric-Coordinates/Barycentric-Coordinates.html **)**

Examples of points for which $w_2$ is positive but the remaining coordinates are negative can also be constructed using this picture; for example, if one draws the perpendicular from $\mathbf{P_2}$ to the line $\mathbf{P_1P_3}$, then each point **S** on this perpendicular which lie above $\mathbf{P_2}$ in the picture (alternatively, $\mathbf{P_2}$ is between **S** and the foot of the perpendicular on $\mathbf{P_1P_3}$) will have this property**.**

***Illustration of a* 2 – *simplex .*** We shall use a modified version of Figure **1;** the points of the **2** – simplex with vertices $\mathbf{P_1}$, $\mathbf{P_2}$, and $\mathbf{P_3}$ consists of the triangle determined by these points and the points which lie inside this triangle (in the usual intuitive sense of the word)**.**

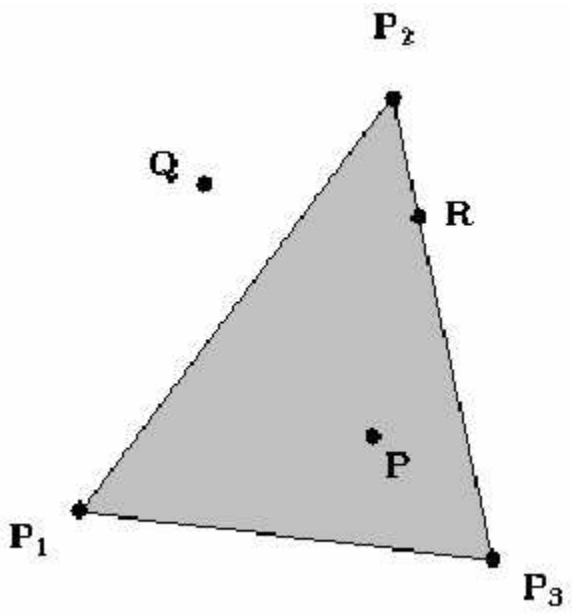
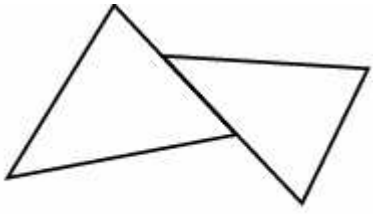



**FIGURE 2**

In this picture the points **P** and **R** lie on the simplex $\mathbf{P_1P_2P_3}$ because their barycentric coordinates are all nonnegative, but the point **Q** does not because one of its barycentric coordinates is negative**.**

Note that the **(*proper*) *faces*** of this simplex are the closed segments $\mathbf{P_1P_2}$, $\mathbf{P_2P_3}$, and $\mathbf{P_1P_3}$ joining pairs of vertices as well as the three vertices themselves (and possibly the empty set if we want to talk about an empty face with no vertices)**.**

***Simplicial decompositions.*** It is useful to look at a few spaces given as unions of **2** – simplices, some of which determine simplicial complexes in the sense of the notes and others that do not**.**

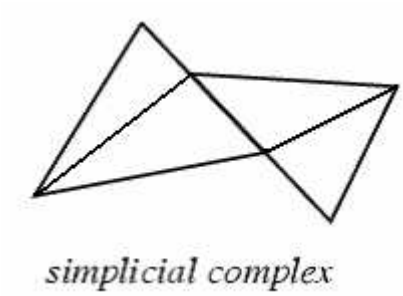


*not a simplicial complex*

**FIGURE 3**

**(*Source:*** http://mathworld.wolfram.com/SimplicialComplex.html**)**

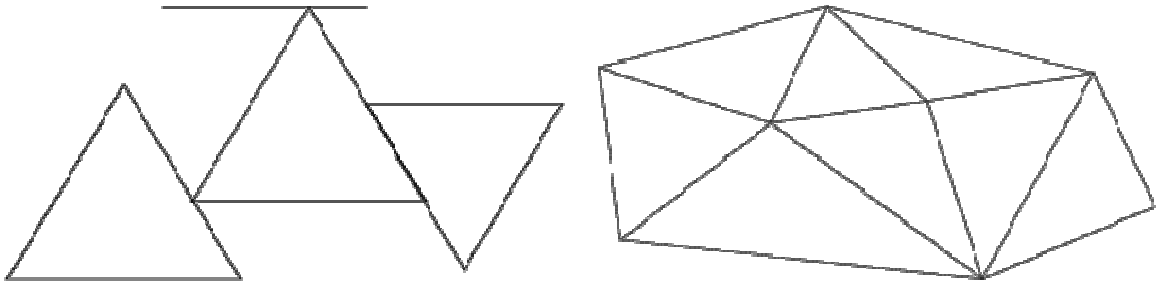
In the example above the intersection of the $2-$ simplices is not a common face. On the other hand, we can split the two simplices into smaller pieces such that we do have a simplicial decomposition**.**



*simplicial complex*

**FIGURE 4**

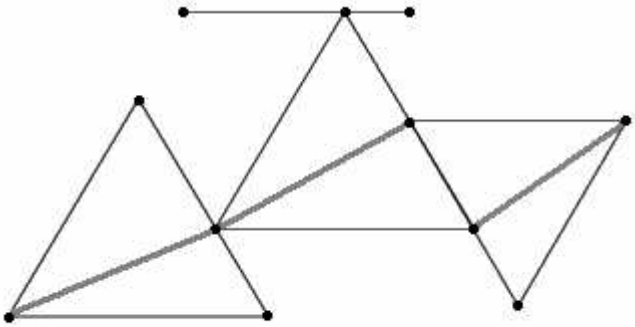
Here are two more examples; in the second case the simplices determine a simplicial complex and in the first they do not. As in the preceding example, one can subdivide the simplices in the first example to obtain a simplicial decomposition**.**



Not a simplicial complex

A simplicial complex

**(*Source:*** http://planning.cs.uiuc.edu/node274.html**)**



A simplicial complex

**FIGURE 5**

***Triangulations.*** In the example from page 523 of Marsden and Tromba, the annulus bounded by two circles is split into four isometric pieces as in the drawing on the next page**.**

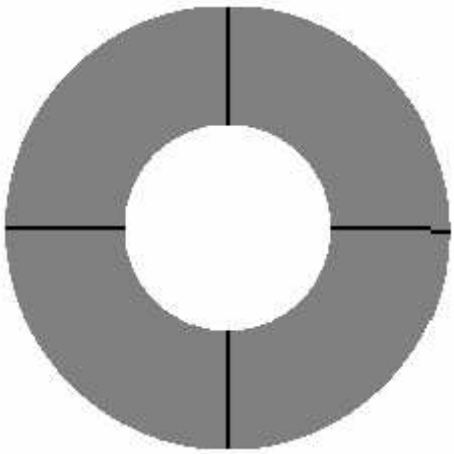


**FIGURE 6**

Each of the four pieces is homeomorphic to a solid rectangle. Since a solid rectangle has a simplicial decomposition into two $2-$ simplices, one can use such a decomposition to form a triangulation of the solid annulus**.**

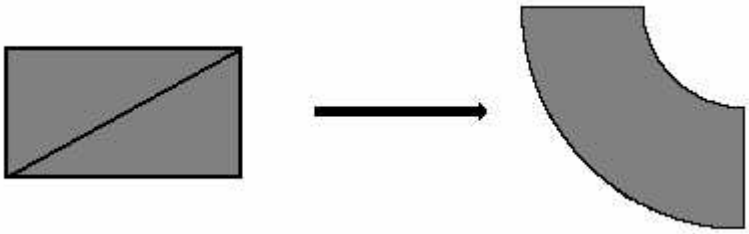


**FIGURE 7**

Another way of triangulating the annulus is suggested by the figure below**:**

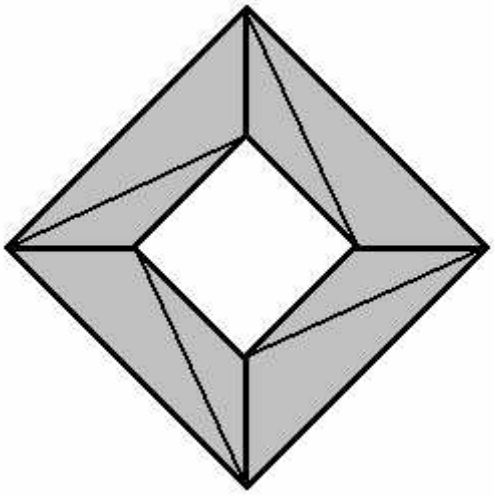


**FIGURE 8**

Similarly, many familiar closed polygonal regions can be triangulated fairly easily. Here is an example for a solid hexagon**.**

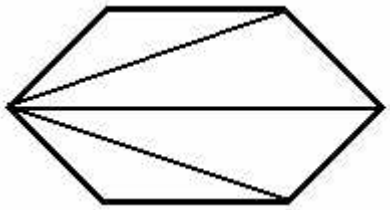


**FIGURE 9**

***Triangulations of prisms.*** The drawings below illustrate the standard decomposition of a $3$ – dimensional triangular prism**.**

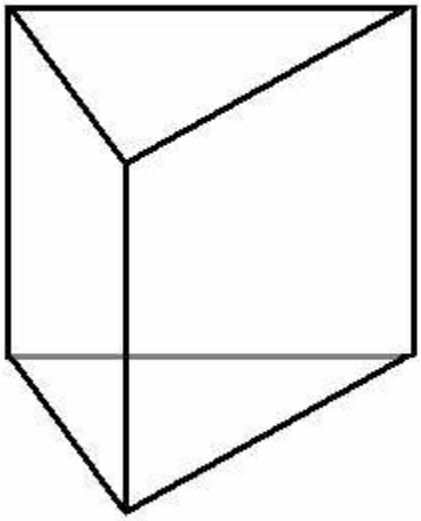


**FIGURE 10**

If we take $\mathbf{x_0}$, $\mathbf{x_1}$, and $\mathbf{x_2}$ to be the vertices of the bottom triangle and $\mathbf{y_0}$, $\mathbf{y_1}$, and $\mathbf{y_2}$ to be the vertices of the top triangle, then the decomposition is given as follows:

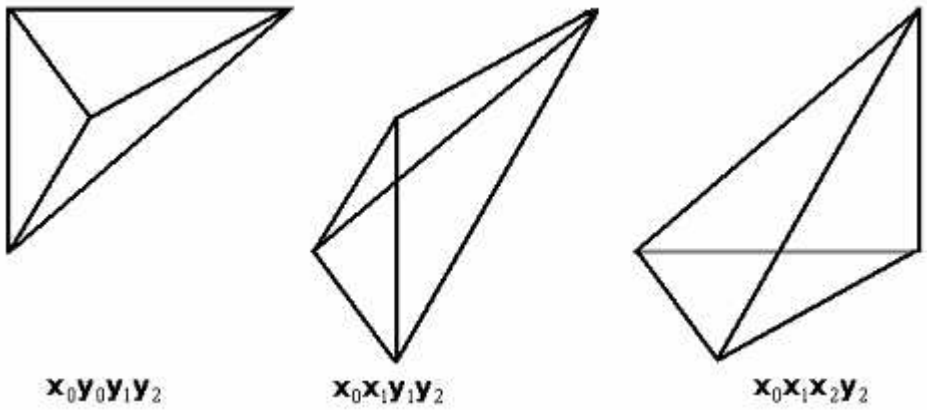



**FIGURE 11**