Mathematics 246B, Fall 2007, Take-Home Examination

Extra credit addendum

The fourth problem on `takehomew09.pdf` will be an extra credit problem, and the following is an additional extra credit problem:

5. [20 *points*]

(a) Suppose that $(P, \mathbf{K})$ is a connected simplicial complex and $f : P \to P$ is a continuous map which is homotopic to a constant. Prove that f has a fixed point. [*Hint:* What is its Lefschetz number?]

(b) Give an example to show that the naïve converse to the Lefschetz Fixed Point Theorem is false; namely, if P is given as above and $f : P \to P$ has Lefschetz number equal to zero, then it does not necessarily follow that f has no fixed points. [*Note:* However, in many cases it follows that if the Lefschetz number is zero then f is homotopic to a map without fixed points; one general reference is E. Fadell, *Recent advances in fixed point theory*, Bulletin of the American Mathematical Society **76** (1970), 1—29 — this paper is freely available online.]