

**Mathematics 246A**  
**Algebraic Topology — I**  
**Detailed Table of Contents**  
**Winter 2007**

**Department of Mathematics**  
**University of California, Riverside**

# Detailed Table of Contents

<b>Preface</b> .....	1
<b>Prerequisites</b> .....	3
Set theory .....	3
Topology .....	3
Algebra.....	4
<b>I. Foundational and geometric background</b> .....	5
1. Categories and functors .....	5
Examples of categories .....	6
Functors.....	7
Examples of covariant functors .....	7
Contravariant functors and examples .....	8
Properties of functors .....	10
Natural transformations.....	11
Equivalences of categories .....	13
2. Barycentric coordinates and polyhedra .....	13
Affine independence and barycentric coordinates.....	13
Sets with simplicial decompositions.....	14
Decompositions of prisms .....	15
3. Subdivisions .....	18
Simple examples .....	18
Definition of subdivisions .....	19
Barycentric subdivisions .....	19
Diameters of barycentric subdivisions .....	22
<b>II. Homotopy and cell complexes</b> .....	23
1. Homotopic mappings.....	23
Special types of homotopy equivalences.....	25
Counting homotopy classes.....	26
Important standard notation.....	27
2. The fundamental group .....	28
3. Abstract cell complexes .....	28
Adjoining cells to a space .....	31
Cell complex structures.....	34
4. The Homotopy Extension Property.....	36

III.	<b>Simplicial homology</b> .....	39
	Some motivation from vector analysis .....	39
1.	Exact sequences and chain complexes .....	40
	Examples .....	41
	Graded objects .....	41
	Chain complexes .....	42
2.	Homology groups .....	44
3.	Homology and simplicial complexes .....	45
	Three definitions of simplicial homology groups .....	45
	Acyclic complexes .....	48
	The Königsberg Bridge Problem .....	50
4.	Comparison principles .....	51
	Application to simplicial complexes .....	52
	The Five Lemma .....	53
	The isomorphism theorem .....	53
	Application to barycentric subdivisions .....	56
5.	Chain homotopies .....	56
	An important example .....	57
IV.	<b>Singular homology</b> .....	59
1.	Definitions .....	59
	Some simple properties of homology groups .....	60
2.	Eilenberg-Steenrod properties .....	61
	The Homotopy and Excision Properties .....	62
	Excision and adjoining cells to a space .....	63
	Equivalence of singular and simplicial homology .....	65
	Homeomorphism types of spheres and Euclidean spaces .....	65
3.	Computations .....	67
	Betti numbers and torsion coefficients .....	67
	Cellular homology .....	68
	Convex linear cells .....	69
	Rational homology .....	71
	Euler characteristics and Euler's Formula .....	73
4.	Proofs of homotopy invariance and excision .....	75
	Homotopy invariance .....	75
	Barycentric subdivision of singular chains .....	76
	Small singular chains .....	77
	Application to Excision .....	78
	Mayer-Vietoris sequences .....	78
5.	Homology and the fundamental group .....	80