The Isotopy Extension Theorem – 2

This is just a footnote regarding the proof which appears in Chapter 8 of Hirsch. The main idea in the proof is a correspondence between diffeotopies and certain types of time – dependent vector fields; the latter are smooth maps from an open neighborhood V of $M \times \{0\}$ in $M \times \mathbb{R}$ to the tangent space $\mathbf{T}(M)$ such that the value at each (x, t) is a tangent vector to x. These are in $1 - 1$ correspondence with vector fields on subsets like V in $M \times \mathbb{R}$ such that under the usual identification of $\mathbf{T}(M \times \mathbb{R})$ with $\mathbf{T}(M) \times \mathbf{T}(\mathbb{R})$ the second coordinate is just the unit vector field d/dt, and not surprisingly the correspondence is given by taking the integral flow of the associated vector field, where we now assume the time – dependent vector field is defined over all some open subset $M \times (-\varepsilon, 1 + \varepsilon)$. The additional condition in Hirsch (bounded velocities) is added to ensure that the integral flow of this vector field will be complete. Actually, one only needs a weaker version of this; namely, it suffices to know that the time – dependent vector field has compact support in the sense that the time – dependent vector field is zero off a compact subset of the form $K \times [-\delta, 1 + \delta]$ where K is (necessarily) compact and $0 < \delta < \varepsilon$. In the spirit of Hirsch's exposition, one can verify that there is essentially a $1 - 1$ correspondence between vector fields on the product manifold $M \times (-\varepsilon, 1 + \varepsilon)$ with such compact supports and ambient diffeotopies of the latter which are stationary off compact subsets of the form $K \times [-\delta, 1 + \delta]$ described above (in other words, the associated diffeomorphism of the product manifold has compact support as defined in the main document on this topic).