$$(z x^p y^r, x^{-b} y^a)$$

because $z^{ap+br} = z^1 = z$ and

$z^{ab-ba} = z^0 = 1$.

Finally, note that

$$S^1 \times_{\mathbb{Z}_a} D(t^b) = S^1 \times_{\mathbb{Z}_a} D(t^{b+ka})$$

because $D(t^b)$ and $D(t^{b+ka})$ are

$\mathbb{Z}_a$-isomorphic.

Finally, we need to specify how one glues on a copy of $D^2 \times S^1$ along (outer boundary circle) $\times S^1$. Given $f \in \mathbb{Z}$, we do this by the S^1-isomorphism

$$(x, z) \longrightarrow (x, x^f z).$$

So we get a manifold with the following data:

$(f;\ a_1, b_1, \ldots, a_g, b_g)$

One can show that the manifold does not depend upon the choices for p and r, but we shall not do so here.

From these data one can present the fundamental group of the manifold as follows:

GENERATORS: $z, y_1, \ldots, y_g$

RELATIONS: $\prod y_i = z^{-f}$

$y_j^{a_j} = z^{-b_j}$ (all j) $\quad [z, y_j] = 1$ (all j).

<u>Note</u> If S^1 acts on X and $h: S^1 \to X$ $h(z) = z \cdot$ base point, then Image $h_* \subseteq$ Center π_1 (X, basept).

See Jankins–Neumann p. 34 for the presentation (but there are notational differences!)