III. PREPARATION FOR A CLASS SESSION

The following sections of Rishel's book should also be read at this point:

- What Goes On in Recitation?
- What Should Be on a Syllabus?
- Lesson Planning: Survivalist Tactics
- Making Up Exams and Quizzes
- Motivating Students
- How to Solve It
- Teaching Methods for Various Types of Classrooms: Lesson Planning
- Mathematical Talks

You should also read the sections in the UCR TA handbook (*The Next Step*) on presentation of [your]self (pp. 41–42), teaching and grading philosophies (pp. 45–48), and classroom logistics (pp. 61–62), and tips for teaching problem solving classes for additional suggestions. In some cases the advice is intended more for humanities or social studies courses and does not apply directly to mathematics courses, but there are still many points worth considering.

Most areas of human knowledge are a mixture of facts and value judgments. Since the ratio of the first to the second is very high in mathematics, an instructor can make statements with a great deal of confidence and accuracy. However, this imposes an obligation for the instructor to know what he or she is talking about and to know when he or she does not. The latter happens to everyone, but one way to minimize the problem is to prepare adequately. Even if students do not know the subject matter, they can often tell if the instructor is unprepared. If you can't answer some relevant question, you should be prepared to follow up with an answer, explanation or referral next time.

Basic segments of a classroom session in mathematics

A class taught by a graduate teaching assistant will contain some or all of these four parts:

1. Information for the students (for example, announcements, distribution or collection of papers and feedback).
2. Time to answer student questions on assigned material.
3. Coverage of new material (in some lower level courses only).
4. Administration of a quiz (but not necessarily always).

For each applicable segment it is necessary to plan **(A)** *what must be done* ***before*** *the class*, **(B)** *what must be done* ***during*** *the class*, and **(C)** *what must be done* ***after*** *the class*.

Allocation of class time

Perhaps the best way to start is to indicate the typical amounts of time to be spent on each

item for 50 minute discussion sessions. Actually there are three basic cases; namely, a pure 50 minute discussion session for a 4 credit hour class or the five credit hour classes Mathematics 22 and 113, a mixed 50 minute lecture-discussion session for Mathematics 5, and a 50 minute primary lecture session for Mathematics 3 (or perhaps other courses for certain advanced TAs).

In the case of *a typical discussion session for a* 4 *hour class with quiz*, the first item should not take much more than 5 minutes and definitely less than 10 minutes at the beginning of class. If there is a quiz, allow about 15 minutes (and definitely at least 10 minutes!). The rest of the class should be devoted to answering student questions about homework assignments or other class material. In the case of *a mixed lecture-discussion section* the following table gives some guidelines:

Activity	Option 1: return quiz	Option 2: give quiz	Option 3: neither
returning papers	10	5	5
homework questions	15–20	15	15–20
new material	20–25	20	25–30
giving a quiz	0	10	0

If there is flexibility for scheduling the quiz (*i.e.*, it is not mandated for a specific day), then a look at the course schedule for a week in advance is recommended to avoid giving or returning the quiz when there is a large amount of new material to cover. In the case of a *primary lecture session,* unless you receive other instructions (*e.g.*, from the Learning Center for Mathematics 3), most of the time should be devoted to covering new material. Students should have an opportunity to ask questions, but in most cases you should reserve the right to limit student questions to a predetermined amount of time like 15 or 20 minutes (you may want to exceed this, but it gives you a point at which you can say it is time to move on). Aside from this the guidelines for mixed lecture-discussion sections apply.

The detailed comments below will probably contain things to displease everyone, but at least they provide a basis for each TA to structure his or her own timetable. Many TAs or instructors find it better to do things in a different order; if you wish to rearrange the segments, you should feel free to do so if the instructor says it is all right.

III(A). What to do *before* class

Information for the students. Determine what needs to be done. Various sorts of announcements may be necessary or desirable, including reminders about important dates, new assignments, the results of a recent quiz or examination, or topics that the students will be expected to know. Look through any papers that are to be distributed

including homework, quizzes, examinations or other written messages given to you by the primary instructor or the Department. Prepare comments on these announcements and papers as needed. **And don't forget to bring all of the necessary papers to class!**

Answering student questions. Read through the homework problems that the students are expected to complete before the class. Choose some problems of each type and group them together in your lesson plans. Write out the solutions to the problems carefully and in detail so that you do not get stuck unexpectedly in the middle of class and you can recover more quickly if some distraction takes place in the class. There will be a more detailed discussion of the mechanics of writing up solutions later, and there will be an example. It is easy to dismiss the homework problems as too easy to require any real preparation, but the consequences of this for both yourself and the students can be very unpleasant. Even though the preparation can be extremely dull and tedious, this is part of the job. Knowing how to do something and being able to do it with a minimum of mistakes in front of an audience are two separate things.

Presenting new material. In those classes where new material is covered, this may well be the most important part of the class at least half of the time. It cannot be skipped or dismissed. Even if students spend a great deal of time reading the material, they are not expected to learn the material by themselves. None of the elementary courses we are considering are even remotely theoretical in nature; their main goals are to teach students how to solve certain types of problems correctly, and the exams generally are closely tied to the homework. There will be a more detailed discussion of the mechanics of writing up new material later, and there will be a fairly lengthy example.

Preparing a quiz. If you put some extra time into advance preparation, you can often save yourself a great deal of unnecessary work in grading the quiz afterwards. It is absolutely necessary for you to write out complete solutions in advance and determine how much time it takes you to do so. It is also important to make at least tentative plans for assigning partial credit. Any such plans will probably need to be modified while you grade the quizzes, but allocation of partial credit also helps you focus on exactly what you want to test on the quiz. As for the other class segments, additional details will appear in the next unit. If you write your quiz out on paper for distribution to the students, check it for accuracy **several times**. It is easy to make minor mistakes that have a major impact on the students' performance, and this is not fair to anyone.

If there is flexibility for scheduling the quiz (*i.e.*, it is not mandated for a specific day), then look at the course schedule for a week in advance and avoid giving or returning the quiz when there is a large amount of new material to cover.

III(B). What to do *during* class

1. Preliminaries. Leave for class soon enough so that you will arrive a few minutes

early. If the preceding class continues past the starting time for your class, you have a right to remind the instructor that it is time for you to start. Begin the class with a clean blackboard; some instructors feel that the board should be erased at the end of a class by the previous instructor, but you cannot count on this and it is not at all productive to become unhappy about this. Besides, leaving the board unerased at the end of class gives students an extra opportunity to copy down things they may have missed. You are under no obligation to save messages left by extracurricular organizations (meeting announcements, Bible verses, *etc*.); if there is enough space left, you can keep them if you wish, but usually blackboard space is so limited that you really cannot afford to do so. This is the best time to write down self-explanatory messages such as lists of answers (but not the solutions) to problems with no answers in the back of the book, examination dates, reminders of assignments that are coming due, *etc*.

2. Returning homework and quizzes, if applicable. Although it is common for homework to be returned by passing the stack of papers around the class, this may not always be the best way to do things. Aside from the distractions this creates, it has been argued that such a method violates students' rights to privacy as defined by Federal laws. If there is also a quiz to be returned, it might be best to give both of them back at the same time. Two ways of returning homework personally are to read off the students' names and to circulate through the class and return homework either by asking for an individual's name or – much better – by recognizing the students. Take a couple of minutes to say what went well and what needs more work without doing any problems. For quizzes one should concentrate on common mistakes and questions that were missed by most students. It is also useful to tell the students what their scores mean.
3. Collecting the homework, if applicable. Some instructors may prefer to do this after answering questions about the homework (the next item). One advantage of this is that students can follow what they did and ask more specific questions. On the other hand, a strong reason against collecting homework after answering questions is that some students will probably copy down your solution as your present it. To avoid complaints and grade appeals, all sections of a course should follow the same policy on collecting homework before or after questions.
4. Answering questions about homework. Frequently this is the most important part of a discussion section. The students' wishes for working specific questions should come before the instructor's preferences. It is particularly useful to have some idea about the number of students who wish to see a specific problem worked. Here are some more specific suggestions on how to organize the answering of homework questions: Before answering any question, (*a*) ask for all the homework questions the students are interested in seeing, (*b*) for each such problem ask how many students had trouble with it, (c) group similar problems together, giving a brief explanation (if possible) of what the problems have in common (*e.g.,* they all involve the chain rule), (*d*) do one problem from each group if there might not be enough time to do everything (you can always go back and do more if there is extra time at the end), (*e*) simply provide hints to get students started on the most difficult problems if you are short of time and the material is of secondary importance. Although many instructors do not poll students about difficulties with specific problems, it does give students

feedback about how many others had difficulties with a given question and it allows instructors to start with questions that interest the largest numbers of students. While answering questions, (*a*) try to learn where students got stuck or what they tried, (*b*) think about interacting with the students while you work the problems, (*c*) give students chances to ask questions, (*d*) don't copy the entire statement of the problem, (*e*) draw pictures, (*f*) try to use the notation and terminology of the book, (*g*) explain what to do and why to do it, (*h*) express the solution in a step by step fashion as much as possible, (*i*) proceed at a reasonable pace, avoiding the frequent mistake of going too rapidly and the less frequent mistake of going too slowly. Suggestions on the mechanics of speaking to the students in front of class and the use of the board will be presented later. — If this is a class where new material is also to be presented, the time spent on homework **must** be monitored carefully to avoid cutting into time required for new material. Inadequate time for new material usually propagates to large numbers of questions at the next class meeting. *Important note:* The classes that include coverage of new material are very elementary, and students in such classes are less tolerant of small mathematical mistakes than students in calculus or more advanced courses. This applies equally to answering questions about homework and presentation of new material. It can seem extremely unfair if the instructor is allowed to get away with mistakes that would cost them points on quizzes and examinations.

5. Covering new material, if applicable. As noted in the previous item, it is particularly important to monitor time carefully in classes that include discussion of homework and new material. Here is a suggestion for a format to use the allotted time efficiently and, for most students, beneficially: (*a*) Check to see which parts of the readings are relevant to the homework problems assigned for the session and stress these portions, and check the book's notation to make sure yours is consistent with the readings. (*b*) Introduce new concepts as much as possible by examples. Instead of copying long definitions from the book, tell the students by an example what the new concept means, giving the definition verbally and referring students to the book for the formal writeup (but do write key words on the board). (*c*) Prepare some remarks and reasons to illustrate the value and uses of the material to be covered. (*d*) Proceed at a reasonable pace, avoiding the frequent mistake of going too rapidly and the less frequent mistake of going too slowly. (*e*) Do not use examples that are worked out in the book, and choose examples that are relevant and relatively short; an example that takes 15 minutes wastes a great deal of time and allows students' minds to wander unnecessarily. — Examples that are too theoretical or too complicated are disasters waiting to happen, and examples taken straight from the book guarantee that the students will be sleeping (if you are lucky) or talking disruptively to each other.
6. Giving a quiz, if applicable. Erase the boards *completely* except for anything that is relevant to the quiz. If you write the problem(s) on the board, do so very clearly and **check carefully for accuracy** immediately afterwards. It is strongly recommended that you do not say anything or give hints during the quiz. Watch the students while giving the quiz and collect the papers promptly when the bell rings at the end of class. If students ask questions about the quiz immediately after class and you would prefer to postpone them until the next class, you have a right to do so.

III(C). What to do *after* class

If it is possible for you to stay a couple of minutes after class to answer short questions, this is extremely worthwhile. Not only can it eliminate the need for short office visits, but it also reinforces an image of concern for the students.

At some point when you have some time to yourself, you should think about the following two basic questions.

1. How well did things go as planned?
2. What has to be done before the next class meeting?

If something did not go as well as you expected, you should make a note of it and think about what might have worked better. You should view this as dispassionately as possible and avoid either blaming yourself or others to any great extent.

Regarding the second question, it is important to know what might have been left undone in the class and make plans to take care of such matters at the beginning of the next class. This may include finding the answers to questions that were raised but not settled. If there is a quiz, of course you need to decide when you are going to grade it. If you make arrangements for students to come to your office outside of normal hours, make a note of this so that you do not forget. If you need to make any adjustments in course records, try to do so promptly because such things are easy to forget.

Guidelines for preparation

It is not unusual for a beginning instructor to become nervous and go too fast. When preparing your notes it is good to imagine that you are in the classroom interacting with the students, complete with delays for questions and side issues. In most cases two of three hours out of class are enough to prepare for each hour in class. Some more specific guidelines will be discussed according to the breakdown for a discussion session described above.

Preliminaries. While grading homework and quizzes, write down common mistakes and misunderstandings. Put the papers together in alphabetical order to expedite grade recording and class distribution.

Answering questions about homework. Know what was covered and assigned since the last discussion session. Write up complete solutions to the assigned problems. If there is a complete instructor's solutions manual it is all right to use it for preparation, but bringing it to class is discouraged. Plan how you will explain solutions, anticipate extra questions, consider different ways of doing problems correctly in case questions arise in this direction, anticipate the time needed for each problem, and think about

how you will use the blackboard. Prepare extra problems in case there aren't enough questions in class. Writing solutions in a two column format, or with two colors of ink – a standard color for the solutions and a less standard color for comments and questions – might be helpful in the beginning. Here is an example of the latter with different type fonts instead of different colors of ink:

888. Find and classify the critical points of $f(x) = 4x^3 - x^4$.

SOLUTION.

- **Ask definition of a critical point.**
- Critical points are points where the derivative is zero or undefined.
- **Emphasize first part of definition.**
- Set $0 = f'(x) = 12x^2 - 4x^3 = 4x^2(3 - x)$. Therefore the critical points are 0 and 3.
- **Ask what to do next. How many used the first derivative test?**
- Observe that f' is positive below zero, and also between 0 and 3, and negative above 3.
- **Ask what this means for the original function f.**
- f is increasing below zero, and also between 0 and 3, and decreasing above 3.
- **What is the conclusion?**
- Since f is increasing to the immediate right and left of 0, the point 0 is **neither** a relative maximum or minimum. Since f is increasing to the immediate left of 3 and decreasing to the immediate right of 3, the point 3 is a **relative maximum**.

Estimated time for the preceding. With the questions that appear in **the special font** this should take 6–8 minutes, and without questions this should take 4 minutes.

Here is another example:

999. Simplify the following expression:

$$\frac{2x+5}{x^2+6x+9} + \frac{x}{x^2-9} + \frac{1}{x-3}$$

ESTIMATED TIME. With questions in **the special font** this should take 6–8 minutes, without the questions 4–6 minutes.

SOLUTION.

Ask what to do and why.

One first needs to find the lowest common multiple of the denominators so that everything can be put over a common denominator. Since

$$x^2 + 6x + 9 = (x+3)^2 \quad \text{and} \quad x^2 - 9 = (x+3)(x-3)$$

it follows that the lowest common multiple is $(x+3)^2(x-3)$. Next rewrite each term in

the expression as a fraction with the latter as denominator.

Ask the students how.

The original expression can be rewritten as follows:

$$\frac{(2x+5)(x-3)}{(x+3)^2(x-3)}+\frac{x(x-3)}{(x+3)^2(x-3)}+\frac{(x+3)^2}{(x+3)^2(x-3)}$$

After expanding the numerators, adding them together and combining like terms (*with numerous details suppressed!!!*) the expression simplifies to the following:

$$\frac{4x^2+8x+6}{(x+3)^2(x-3)}=\frac{2(2x^2+4x+3)}{(x+3)^2(x-3)}$$

Is this all? Why? What if the numerator factors?

A discussion of problem solving is not complete without some mention of the classic book, *How to Solve It*, by George Pólya. His eight steps for solving a mathematical problem form an excellent checklist when preparing problems for class, especially if the problems are challenging for the students.

1. Read the problem.
2. Read it again.
3. Draw a picture or diagram.
4. Find and label the unknowns that you are looking for.
5. Find and label the known quantities.
6. Write down all formulas and relations between knowns and unknowns.
7. Solve the problem
8. Check the answer.

As noted in Rishel's book, one could add one more item to this list for classroom purposes: Think about how one might generalize the problem.

Covering new material. We begin with some general comments. One of the main problems of beginning instructors is going too fast or too slow. As noted before, it can be helpful to imagine that you are speaking to the students while preparing. There is a natural tendency to treat the subject matter of lower level courses as a sequence of tricks for solving problems, and to help counteract this a teacher should try to point out the unity and reasonableness of the material. However, one should not go too far. The section of Rishel's book on motivating students discusses some possible examples. Explanations are generally better than actual proofs in lower level courses; the ability to use the material in other subjects and to think independently are the main goals for most students. Students should learn how to analyze problems and

understand what a reasonable answer should look like. Finally, one of the best ways to make lectures more interesting is to show an interest in the material yourself. Announcing in advance that a major topic will be covered can reinforce the structure of the course to the students and provide a sense of accomplishment when you are done. For each topic in the course there are four important time settings:

1. This will be covered soon.
2. We are now covering this.
3. Now that we have covered this, we can proceed.
4. You will be responsible for this.

Here are some specific points. If the course outline indicates that a topic is to be covered over several sessions, make sure of what you will do in each class session and what homework will be due at each meeting. Don't forget to give the homework assignment to the students. Look at the problems assigned for the next meeting and choose good examples. Read the section to see how the book introduces the new concepts or solves similar problems, and skip or de-emphasize the parts of the readings for which no homework is assigned. Write down the solutions for the examples you choose, not so much to make sure you can do them but rather for the reasons listed for doing assigned homework yourself. If an example takes too long for you to work or does not seem quite as useful as you first thought, discard it and choose another. Prepare additional examples. For purposes of budgeting time it is useful to start your notes with an allocation of time and a listing of the time required for each example. If time becomes short, you can then decide quickly which of the remaining examples can best be deleted to keep within your time constraints. During lectures on new material, students will be less involved that during time spent on answering homework questions. However, some involvement of students is very useful in providing encouragement and feedback to students. For example, before introducing a new concept, you might ask how many students had seen it before and what they remember about it. This is more effective with concrete examples (*e.g.*, how many know how to factor the simple quadratic $x^2 - 5x + 6$ rather than how many know how to factor polynomials). Such feedback is the only way to see if the pace and level of presentation are appropriate for the students. Finally, look back at the examples you prepared, check whether everything can be done within the time available, and think about what you will cut if you run out of time. Write all this down so you don't have to figure it out under pressure in front of the class. Ideally a lecture should complement the text, providing a more intuitive and informal account of the material, including things that one might not choose to say in print. Lectures are most effective when students have already tried to learn the material, so classes should be strongly encouraged to read the text before and after the lecture. — You need to keep track of the amount of time left for presenting new material. Starting new topics near the end of the time is not recommended because there is a natural temptation to run over the allocated time and in any case it is extremely likely that everything will have to be repeated in the next class.

Giving a quiz. If you need to choose your own problems, choose examples that are similar to those in the homework or modify some assigned problems. In all cases write out complete solutions in advance and time yourself; it should only take you 4–5 minutes to do a 10 minute quiz. Giving students 15 minutes to do something that should take only 10 minutes is reasonable, but giving them 15 minutes to do something that should only take 7 minutes does not prepare them well for examinations. See the subsection "Designing and grading quizzes" below for more detailed information on in-class quizzes.

What if there are too many or too few homework questions?

Homework assignments vary in length and difficulty, so you have to expect that sometimes there will be more questions than can be fit into the allocated time and at other times there will not be enough questions. We have already discussed methods for dealing with an excess of questions. On the other hand, even if there are not enough questions it does not necessarily mean that students have mastered the material, so in these cases one needs to work with the students to get things moving.

1. ***What if there are not enough homework questions?*** If this happens occasionally, you might want to pick out some problems that you expected the students to ask about and ask them specifically if they have an questions about these. This often encourages students to speak up. If the answer is no even after some gentle prodding like, "If I gave this as a quiz problem, then everyone would be able to do it?" don't work the problems; it will only bore them and encourage discipline problems. Instead, check from your solutions to see if some problems deserve interesting comments (for example, one where the second derivative test fails) and ask questions directly related to the comments. Another option that some instructors might like is to give the class extra problems to work on, having the students work in groups, offering help when needed, and getting volunteers to explain the solutions at the blackboard. One further possibility, which would require the primary instructor's approval, would be to pass out short quizzes that are graded in class; this usually generates questions. Early dismissal could lead to a situation where there are routinely no questions or there is peer pressure among students to refrain from asking questions.
2. ***What if there are too many homework questions?*** As noted before you need to know how many problems you want to do before you start and you need to choose the problems you will work. Here are some further suggestions beyond the earlier comments: Stress the fact that the problems not done are similar to those that were done or explain in less than a minute what the difference is. Encourage the students to try the problems again and to go to an office hour for further help.

What if the attendance in discussion sections is poor?

This should be reported to the primary instructor.

Special considerations for primary lecture sessions

Of course there are many similarities between regular lecture sections and ordinary classes where a faculty member is the principal instructor. There will be references to the description of discussion sections for things that are the same or nearly so. We shall begin with some qualitative differences.

1. **YOU** are in charge of covering the course material. — On the other hand, in this sort of standardized course the choice of material belongs to the faculty, and most of the day to day management for Mathematics 3 is handled by the Learning Center. Instructors for courses receive many communications that are not passed on to the teaching assistants; as a primary instructor, your will now receive these sorts of communications.
2. Your relation to the students will probably be somewhat more distant. – Many students perceive the main instructor in a lecture-discussion course as the bad guy and the TA as the good guy. Thus you might end up being the bad guy if you are teaching your own course; however, since the content of elementary courses is highly structured, it is appropriate to mention this fact (without complaining!) to the students so they have less reason to view you as the bad guy.
3. Some undergraduate students might think that you, as a graduate student, are not really in control of things or not capable of teaching them, and occasionally even irrelevant physical traits like gender, ethnicity or size may be factors in some students' minds. — It is important to defuse sentiments of all these types by projecting a confident and professional image. Be well prepared, do not be overly friendly with your students, and if you are teaching the course for the first time do not tell them so. Stick with course policies for special considerations (like scheduling makeup quizzes) and grade assignment. Never show anger or frustration. If you do not know how to deal with disciplinary problems, ask for help as soon as possible.
4. The student population is different. — The levels of courses taught by TAs on their own are generally lower than those in which TAs have discussion sections. Most students are in such courses involuntarily and have seen the material before in some form; it is understandable that some students will not be particularly motivated under such circumstances. Often students in these courses do not like mathematics and find it very difficult.
5. Small mathematical mistakes by the instructor have a greater negative impact. – A few occasional small mistakes in calculus discussion sections generally do not upset the students too much (although frequent ones clearly do!), but as the course level decreases the students become increasingly less tolerant. As noted in the previous discussion, it can seem unfair if the instructor is allowed to get away with mistakes that cost them points on quizzes and examinations.

Unless indicated otherwise (*e.g.*, through instructions from the Learning Center for Mathematics 3), most of the time in such classes should be devoted to covering new material. Students should have an opportunity to ask questions, but you probably should reserve the right to limit student questions to a predetermined amount of time like 15 or

20 minutes (you may want to exceed this, but it gives you a point at which you can say it is time to move on). There is not much that can be added to the comments on the individual parts beyond those made for mixed lecture-discussion sections, but in pure lecture sessions you also have the primary responsibility for passing along important class announcements.