V. INVOLVING STUDENTS DURING CLASS

The need for some student participation in discussion sessions and lower level mathematics classes has already been noted. For example, student input should be the main factor in deciding which homework problems to work in class. Furthermore, asking students questions while going through the solutions to homework problems provides valuable feedback on the points where the students got stuck or went wrong. The following example from last quarter illustrates how an instructor can prepare material to provide opportunities for student input.

999. Find and classify the critical points of $f(x) = 4x^3 - x^4$.

SOLUTION.

Ask definition of a critical point.

Critical points are points where the derivative is zero or undefined.

Emphasize first part of definition.

Set $0 = f'(x) = 12x^2 - 4x^3 = 4x^2(3 - x)$.

Therefore the critical points are 0 and 3.

Ask what to do next. How many used the first derivative test?

Observe that f' is positive below zero, also positive between 0 and 3, and negative above 3.

Ask what this means for the original function f.

The function f is increasing below zero, also increasing between 0 and 3, and decreasing above 3.

What is the conclusion?

Since f is increasing to the immediate right and left of 0, the point 0 is neither a relative maximum nor a relative minimum. Since f is increasing to the immediate left of 3 and decreasing to the immediate right of 3, the point 3 is a relative maximum.

End of solution.

The purpose of this unit is to go further into some reasons and possible methods for involving students. Although it seems likely that some student participation is needed in all lower level mathematics courses and some discussion sections, it also seems clear that not all instructors will be comfortable with some of the suggestions discussed here. However, even if the specific advice in this unit does not seem applicable, it can still be a useful source of ideas for everyone in planning classes.

At this point it is assumed that you have no problems with the day to day running of the classroom, including no (major) problems with structuring a class meeting or dealing with

discipline problems. If you have such problems, these should be overcome before working further on involving students.

The suggestions given here might not always agree with the course policies for a specific course. Such policies and instructions from primary instructors or course coordinators have precedence over the comments appearing here.

The mechanics of involving students during class do not vary much for various assignments such as conducting discussion sessions or teaching courses below calculus, The major difference consists in the amount of time available. In general more time is available during a discussion session in a calculus course than in a precalculus course where one has to cover new material. There is usually more time in courses with stronger students.

In the preceding example, measures to involve students during class were incorporated explicitly into the preparation process. This can be rather tedious, but it is generally unavoidable. Attempts to involve students during class are at best very difficult to do on the spur of the moment. This should be kept in mind throughout the unit.

Why get students involved during class?

One of the simplest reasons for promoting student participation during class is that it makes things less boring for everyone. Although homework, quizzes and exams also meet some of the basic goals of prompting students to think about the subject matter and to let the instructor know what they have learned, it is sometimes the case that little or no homework is graded, and both quizzes and exams penalize a student heavily for what has not been sufficiently understood or assimilated.

Giving and receiving feedback

- Students' responses in class give the instructor ***and*** the students feedback about which parts of the material have been understood and by how many students. Moreover, this happens in a nonevaluating setting, as opposed to quizzes and exams. It also informs the students how they are doing compared to the rest of the class. This sort of information is not otherwise available to the students except if they ask you directly during office hours. This is one reason that one should always ask for all the homework questions and the number of students wanting to see each of them before answering any.
- Returning quizzes is also an excellent opportunity for providing feedback; for example, "the average is ... ," or "nearly everyone could do ... ," or "the following was a common mistake … ." When returning exams given to more than one section, the instructor can and should give feedback on how this section did compared with the other sections; for example, "this section's average is ... while the overall average for all sections is ... ."

Opportunity to redirect

- Involving students allows the discovery and correction of misunderstandings that students may have, again without penalizing the students. This is extremely important when one considers that many of the students taking precalculus — or even calculus — have already had this material in high school. Not only have those students not learned the material well enough to test out of the courses, but often they learned or remember the material incorrectly (the mistaken conclusion that $(a + b)^2 = a^2 + b^2$ is one of the best examples, which comes up in many disguised ways in nearly all lower level courses).

Having a chance to bring common mistakes to the students' attention after one (or several) have made them is much more constructive than just telling the students that those are mistakes they should not make. Incidentally, it is not a good idea to in low level courses for instructors to make mistakes on the board intentionally; many students will be confused and frustrated.

Pacing the class; the only possibility of tailoring the course somewhat to the students' needs

- Students asking questions and students answering questions both give the instructor feedback about the pace and level of the class. Though we do not have the luxury of stopping and redoing a lecture because some students did not understand something (especially in precalculus courses), there is some leeway. Frequently there is an opportunity to do an extra example, emphasize or repeat something when it is clear from the interaction with the students that this is needed. Or even promise that you will certainly do one of the homework problems on that topic next time if they still have difficulties then. If one does not find out until later (like after a quiz or exam) that certain topics were not sufficiently understood, it usually takes much longer to fix the problems.
- Conversely, if it is clear from the students' responses that a certain point has been understood by most, there is no reason to do three more examples or explain things again.
- In particular, going over the homework problems in any course can, and in fact must, be tailored to the students' questions in all courses. As noted before, one should always ask for all the questions and how many students want to see each before answering any.

Fewer discipline problems

- A fair portion of discipline problems is due to students who are bored and uninvolved during class. There is nothing for them to do but listen and take notes, so why not talk to their neighbors.

Facilitates learning

- In the long run, we expect the students to have learned to solve certain problems by themselves. Watching someone solve problems and solving problems on one's own are two very different things. Think about the different levels of learning as points in the interval [0,1] where 0 is watching the instructor do problems on the board and 1 is the students doing problems on their own with their books and notes unavailable. The former sometimes helps with the latter, but there is a big difference in level between the two. Moreover, students are tested on the latter. Involving students during class introduces a step between the two; namely, students solving problems with (a lot or little) help from the instructor. It can be compared to learning to ride a bicycle; watching someone ride can be helpful but it is a big step from riding one yourself. Having someone hold the bicycle while you try to ride it yourself is helpful and less painful. One probably oversimplified way of putting things is to say that students may well forget if they are simply told how to do an example, they will probably remember how to do an example if they are shown how to do one, and they are most likely to understand how to do an example if they are involved in doing one.
- *Active learning.* Studies in biology suggest that "learning" (*i.e.*, the brain processing and storing information in such a way that it can be retrieved and used) is facilitated by active participation of the person. For example, most people learn fewer words from a list that is

read to them if they do not do anything than if they repeat the words to themselves after hearing them.

Encouragement for students

- Answering questions correctly can boost a student's self confidence, something that students in lower level courses often need badly. Even if a student answers a "good" question incorrectly, this can be an opportunity to encourage him or her. More will be said about this later.

What you can and cannot expect (or reasons given for not involving students)

Some valid concerns are what happens if no one asks any questions, if no one answers your questions, if the students ask a lot of questions about things they should know, as well as, "I tried it once but it didn't work," and, "`When I try asking questions I cannot cover the material." We shall discuss some possible ways of dealing with such problems here.

It is too time consuming

It always takes more time to do an example or homework problem with students contributing to its solution than without such contributions. When time is scarce you need to be aware of this and prepared for it. When you prepare your class, you will want to write down a solution for each assigned homework problem and example you might do in class. The reason for this is not so much to make sure that you know how to do the problem or example, but rather

a) to include some of the previously listed goals (such as feedback and redirecting) of involving students,
b) to think about possible questions to ask (*i.e.*, how to achieve the goals),
c) to think about what questions the students might ask — another form of involvement that might occur,
d) to anticipate where misunderstandings might arise,
e) to have an idea of how long it will take you to go over the problem or example with and without involving students.

Suppose you find that you can do four examples without involving students but only two with involving them. Then you might want to look for a compromise; one without involving them, one with involving them a lot (maybe for feedback and redirecting), and one with some easy questions (for encouragement). Though you might not have gone through as much material, it is likely that the students learned more.

Students are not interested in the course

One cannot expect the students to be involved because they are interested in what is being taught. Most of the students in lower division courses are not interested in mathematics and will never be. They are not mathematics majors but are required to take a certain number of mathematics courses for their majors. That does not mean that they can't or won't do well or that they won't be involved during class, especially if they understand why the class is important.

One attempt at involving students was unsuccessful

It is to be expected that involving students once will not work. Involving students requires a consistent effort from the first meeting on. It is unlikely that the students will start becoming involved if for three weeks they were not expected to do so. Even in a class with a lot of student involvement there will be periods when it seems to stop, especially after an exam or near exam times in their other courses. Perhaps trying a different method or a different combination of ways of involving students will work better with a particular group of students.

I told them that I wanted them to be involved but ...

There is little reason to expect that students will believe that the instructor really wants them to be involved without explaining the reasons to them. Similarly, there is also little reason for the instructor to expect will the students to trust that he or she means it right away; after all, the students don't know the instructor. Here is a response from an instructor with a strong dedication to student involvement: "I usually explain on the first day of each semester in all my classes that I will be asking a lot of questions all semester long and that I expect them to answer them, that often a frown, a nod or shake of the head will be enough of an answer. I tell them that I do this because I need the input in order to pace the class, and that otherwise I tend to speed up. I tell them that I would rather get wrong answers now than on the exam, and that wrong answers are often even more helpful than right ones. I tell them that my memory is limited and though I probably will remember whether a lot of students had problems with, say, the chain rule, I do not even want — let alone can — remember who gave me an incorrect answer."

Various reminders to the students of why involvement is wanted will be needed throughout the term.

The students do not want to be involved

It is easy to say one thing but unconsciously to act as if the opposite were true. The instructor needs to show by actions that he or she really wants the students to be involved. Sometimes it can be as simple as using "we" rather than "I" or stopping and facing the students. When questions are asked, it is necessary to give the student(s) time to think and to formulate an answer. Being quick at answering one's own questions gives a strong signal that student involvement is not wanted, and it leads the students to view all questions as rhetorical ones. Also, a student's answer should be acknowledged even if it is wrong.

It is important to remember that students seldom answer very general questions such as, "How does one factor a polynomial?" but are more willing to give the question, "How does one factor $x^2 - 6x + 5$?" a try. Along the same lines, the question, "Who needs more help with understanding this particular step?" is more likely to get a response than, "Are there any questions?" Rather than asking big, vague, open-ended questions, it is better to link the current problem back to a previous example if possible. For example, "Problem [**blank**] was similar to this; what was the first step that we did there?" Breaking questions into smaller steps also helps. All of these points indicate a need to keep the questions simple, but it is also important to avoid asking insultingly simple questions like, "Who can tell me what $2 + 3$ is?"

Similarly, it is much better to avoid asking questions about an example which is in the book (unless the question is clearly about something not written down in the book) and to use instead problems which are similar to assigned homework; this increases the chances that students will pay attention and participate.

When students ask a question, the instructor needs to deal with it. It is often hard for the other students in the class to hear questions asked (or answers given) by other students. One way of dealing with this is to repeat the question, and at the same time it may be worthwhile to ask at the same time if someone else can answer it. There are two warnings about the latter advice. First of all, questions about course policies and procedures have to be answered by the instructor. Second, when redirecting the question, it is necessary to be careful to avoid giving the message that the instructor does not know the answer.

Not every question should be answered during class. Questions on basic prerequisites, questions that have already been answered several times in the class or only involve a few students, should be acknowledged but not answered (again) in class. "Let us look at it together after class or during an office hour," is a better way to deal with such questions. It gives the student the message that it is now his or her responsibility to put in some extra effort to get them answered.

Trying to involve students when going over a quiz or graded and returned homework is often not constructive. After all, the students should be able to tell from the grading whether something is right or wrong and even what they should have done. Going over the quiz, however, may be a great opportunity to give the students feedback ("The average is 7 and if you received less than 5 you might want to come and talk to me."), encouragement ("The first question is hard and nearly everyone did it right."), and motivation to be involved ("We did a similar one ***together*** in class and that probably helped."). Mentioning briefly what went well and badly on returned homework (***without*** doing any problems) is also useful; of course, this does not apply for sections where homework problems are systematically answered after the homework is graded and returned.

The instructor needs to earn the students' trust that it is OK to ask questions, to give the wrong answer, that the instructor really does not hold making mistakes against them or does not even remember who made a mistake. If such a mistake is mentioned, one might proceed by saying that someone made the following mistake while, say, pointing to the right side of the class when the instructor knows for sure that it was someone on the left side, and adding that this gave us an opportunity to re-emphasize that ...

They were involved but now they are not

One cannot expect all the students to be involved all the time, and one cannot even expect all the students to be involved some of the time. On the other hand, it is reasonable to expect that most students will be involved some of the time. It is not advisable to let a few students answer questions all the time. Even though it is nice that someone is answering, it can give the rest of the class a message that the instructor is not really looking for ***class*** involvement. More will be said about this later.

Occasionally, some students might put another student down for making a mistake or asking a particular question. It is important for the instructor to make it clear that this view is neither shared nor acceptable (without being too heavy handed in doing so). Otherwise very few if any students will dare to participate.

Techniques for promoting student involvement

As noted in the preceding discussion, being involved is not the natural state for most students, and it requires persistent and consistent effort on the instructor's part to get most students involved most of the time. Furthermore, it is extremely helpful to incorporate the whens and hows (which depend on the whys) or involving students into the preparation for a class meeting. For example, when doing maximum and minimum problems in calculus, one might want to ask the students for the definition of a critical point since they often forget to include the part, "and points where the derivative does not exist." In that case the goal is to emphasize this and, if necessary, redirect; a feedback component can easily be added if the wrong answer is given by asking, before correcting it, how many students agree with the given answer.

Although this section tries to answer the question, "How does one go about involving students?" there is unfortunately no universal answer. Tricks that work well for one person might not work well for another, and it is important for each individual to determine what works and what does not by trying different things. Also, things that work for a first year calculus class might not work for a precalculus class, and even things that work in one section of a course might not work in another section of the same course. Therefore from time to time it is worthwhile for an instructor to try some method that might not have worked in the past.

The method for involving students also depends strongly on the objective. Is it more feedback? Is it reinforcement of something already seen? Is it helping the students discover something for themselves? Even if the goal is established, the most effective method often depends on the specific goal and the group of students in the class.

The suggestions below have been seen specifically and successfully in classroom situations. It is worth remembering that a combination of a couple of the methods might be more successful than just one. For example, the combination of asking a specific question to the whole class (or, if necessary, from a specific student), following up with feedback questions (such as how many agree or disagree with the response), then following up by asking someone who disagreed, usually works better at attaining most goals for involving students than using just one of the three approaches. Also, sometimes switching to another method is necessary to break the monotony. Being asked to raise one's hand every 30 seconds becomes so tedious that students will stop responding. Giving them a problem to work on by themselves or in groups or asking someone to come and do a problem at the board might be a welcome change.

Warmup questions

Sometimes an instructor will write a true–false or multiple choice question on the blackboard before returning the homework. Students quickly learn to expect a quick hand count when class starts on what they think the answer is. Such questions can be about some mistakes or misunderstandings that were found on quizzes or homework or to emphasize (again) or review something that was done earlier. It should take less than a minute to take a poll and then briefly explain the correct answer without actually doing the problem. If it takes longer, the problem was not appropriate for a warmup. It can be a great way to get students started, to get feedback, and to deal with some misunderstandings and to give encouragement.

Asking specific questions of specific students

"A, what would you do next?"

"B, do you agree with the answer of C?"

"C, what is the derivative of $\cos x$?"

Of course, often A and B do not have any answer at all and the instructor is stuck. Or C gives the wrong answer and the instructor ends up putting this student somewhat on the spot.

Asking questions of the given type from students whom you know can answer them but rarely volunteer is often a good way to keep the meeting going while there seems to be student involvement. Resorting to this occasionally is worth considering. Doing it a lot lets the other students off the hook, does not give you much feedback, and does not give the other students who (thought they) knew the answer a chance to participate. Another drawback about asking question of specific students is that most students do not like to be singled out in class. On the other hand, asking such questions from a student who is talking too much with his or her neighbors might be a good way to stop the talking.

Asking specific questions from the whole class

This kind of question does not put anyone in particular on the spot and gives everyone a chance to answer. Two common problems with it are that nobody answers or that the same people always answer. The latter reflects one basic disadvantage of the method: There is even less chance to discover misconceptions by this method than the previous one because often only the students who know the answer will respond.

If no one answers, some options are trying to reformulate the question, asking a smaller or easier question, linking it to an example given before, or giving a hint. There is reason to hope that someone will eventually get tired of this and will volunteer an answer. Maybe the students need a reminder as to why the instructor is asking questions. On the other hand, an instructor should also ask himself or herself if perhaps in other ways he or she seems to imply that he or she does not really want the students to answer the questions.

If the same students always answer (first), an option might be to tell the class that the instructor does not always want the same people to answer, or perhaps that this question is for the back of the class since nothing has been heard from them for a while. Another way to deal with this is to let the same student answer once more but to follow it up with feedback questions as described below. This can be done independently of whether the given answer was right or wrong. Typical questions of this sort would be: "How many of you agree?" or "How many disagree?" or "What did you think the answer was?"

Feedback questions

Questions such as, "How many of you want me to do this homework problem?" or, ``Raise your hand if you think this is an inflection point," do not put anyone too much on the spot. It is the quickest way to get a lot of feedback both for the instructor and the students themselves. Note that it is nearly the only way for a student to know whether he or she is one of many, or one of only a few, who does not know something. Scores on quizzes and examinations do not convey that information. Students usually answer feedback questions, especially if a shortage of hands might

mean that the instructor might not do that homework problem or might go faster. A reminder that there is not enough time to answer all the homework questions, and it is necessary to concentrate on those requested by the most students, is often all it takes.

Answering homework questions probably should always include a feedback component as to how many students want to see each of the questions requested, even if time management is not an issue.

As mentioned earlier, this kind of question can also be very useful even if the same students always answer. Once a student has committed himself or herself to not agreeing with something, it is easier to get that student to say what he or she thought the correct answer should be.

Drill questions

When the instructor wants to get the message across that a certain fact or formula is important and that they should know it by now, using a drilling technique can be helpful. For example, asking, "The derivative of $\cos x$ is ... ?" or saying, "$a^2 - b^2$ is equal to ... ," and it gives the students a clear message that by now a certain fact or formula is supposed to be known. This can become noisy but it gets the point across.

Have a student come and do a problem at the board

Here too one can ask for volunteers or ask a specific student to "volunteer." This approach does not work very well for some people, but it does work extremely well for others. The dangers are quite clear: Nobody volunteers, the person you ask refuses, or the person you ask agrees but it is a disaster. Usually, only students who can do the problem will volunteer, and seeing them do the problems without trouble depresses those who cannot do the problems.

Students often cannot explain their work well, so the instructor must be ready to go over the solution and make sure everyone understands the steps.

If this method is used frequently, it is worth "training" the rest of the class. The other students can and should ask questions of help the student who is at the blackboard. In some classes this leads to a very lively and constructive class interaction with the instructor playing a very small role.

Having someone work at the board is relatively time consuming, especially for a new variety of problem. It might be better to wait until one has a feeling about the students and the class before trying this method.

Give the students a problem to work on themselves

This method is especially useful when the students have no questions the instructor wants to do extra problems or review problems. When the students work together of in groups on the problems, the instructor can go around and see if they know how to start, *etc*. This gives a lot of feedback and can be a pleasant change from the usual "watch and answer questions" routine. This always seems to take longer than anticipated, so it is important to make sure the problems are not too long. At the end, which need not be when everyone is done, the students should receive feedback: For example, "Almost everyone knew how to start," or, "When getting to this point several people wrote ... ." The students should also get a final answer if the problem is not finished at the board.

It is important to be aware of frequently occurring problems with having students work in groups. Just the physical aspect of having the students break up into 7 or 8 groups of 5 people can easily take 5 minutes. Moreover, the students are not used to working with each other unless they do it frequently using the same group structure. Balancing groups as to abilities and the like can be helpful, but even then some people just do not work well together. It frequently happens that one person in a group takes over and imposes his or her answer on the others, independently of whether he or she is correct.

Self tests (more appropriate for precalculus)

The instructor writes down a problem and its answer (not its solution) and tells the students to solve this problem within a certain amount of time on their own at home with their books and notes closed. This is not strictly speaking a way to involve students in class, but it can lead to other involvement the next meeting, generating feedback and questions for next time or encouraging students to get help early on. One can also go over the solution the next time in class if a lot of students cannot do the problem. The method can also be used as a way of stating a goal for a class meeting if the problem is stated at the beginning. "At the end of class, you should be able to do the following type of problem ... ." If time is left at the end of class, one can end by checking how well the goal has been reached.

This method should not be used as a substitute for quizzes.

ACTIVE LEARNING IN LARGE LECTURES

Certainly it is easier to involve students in (relatively) small discussion sections than it is to do so in large lectures. However, some primary instructors may employ a variety of techniques so that students are actively engaged in working with course material in large lectures rather than just passively viewing and hearing it. The campus Center for Teaching Excellence has held panel discussions and all-faculty meetings on teaching large classes during the past few years, and the handouts from these meetings include numerous sections on promoting active student learning in large lectures. Some of the points raised in these handouts will be summarized here. The purpose for doing so is to provide some background in case primary instructors decide to employ such techniques.

One frequently used method of involving students in large lectures is to introduce cooperative or group learning projects. This can be very beneficial, but it can also fail miserably. If this technique is used, good preparation is essential. Clear, simple and relevant instructions are needed, and it might also be good to provide a worked out example. Problems with such projects often arise from the nature and the structure of an activity, so it is important to look very critically at what might go wrong. When such projects are part of a lecture, order should be maintained until the group discussions are scheduled to begin. Firm deadlines for completion of the work are essential; five minutes is recommended, and it seems that ten is an outside limit if only one such project is assigned in a 50 minute lecture (at most two projects of no more than five minutes in such a lecture are advised). The sizes of the student groups should be reasonably small, and each group needs a facilitator to keep the discussion going, a recorder to write down what takes place, and a reporter to present the results to the class. Time constraints may mean that only a handful of groups can report in a lecture. It can take time for instructors and students to get used to group work, so some disappointment with a first attempt should not be overly discouraging.

The handouts also discuss a variety of methods for getting written student feedback in large lectures, including in-class assignments to submit written questions, to write a one minute summary of something covered in the lecture, or to make suggestions for test problems. Other methods suggested in the handouts include choosing groups of students to lead discussions on course issues (with participation possibly part of the course grade) or meeting with groups of students outside of class to discuss course related issues.

Clearly many details need to be addressed in order to carry out any of these ideas, and the details often depend upon the course content and the personal style of a primary instructor. I have copies of the handouts if you are interested in learning more about these matters.