

NAME: _____

MA 511, D04 S01, Fall 1994, Examination 1

The point values for problems are in parentheses.

1. (10 points) For one of the values $c = 10, 12$ the system of equations

$$\begin{aligned}x + 2y + z &= 4 \\2x + y + z &= 2 \\4x + 5y + 3z &= c\end{aligned}$$

has no solution. Determine the value for which no solution exists. [*Hint:* $2(1, 2, 1) + (2, 1, 1) = (4, 5, 3)$.]

2. (15 points) Let V be a vector space, and suppose that $\{v_1, v_2, v_3\}$ is a linearly independent subset of V . Show that $\{v_1 + v_2, v_2, v_3\}$ is also linearly independent.

3. (15 points) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

4. (20 points) Express the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 6 & 5 \end{pmatrix}$$

as a product LU where U is upper triangular and L is lower unitriangular.

5. (10 points) Suppose that A is an $m \times n$ matrix, B is an $m \times 1$ matrix, and X_1 and X_2 solve the matrix equation $AX = B$. Show that their average value $\frac{1}{2}(X_1 + X_2)$ is also a solution.

6. (10 points) Is there a matrix A of the form

$$\begin{pmatrix} a & b & 1 & 0 & 0 \\ c & d & 0 & 1 & 0 \\ e & f & 0 & 0 & 1 \end{pmatrix}$$

whose row space is 2-dimensional? Either provide an example or explain why such a matrix cannot exist.

7. (20 points) Find a nonzero vector in \mathbf{R}^4 that is orthogonal to each of the vectors $(1, 2, 3, 6)$, $(0, 1, 2, 3)$, $(0, 0, 1, 2)$.