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MA 511, D04 S01, Fall 1994, Examination 1

The point values for problems are in parentheses.

1. (*10 points*) For one of the values $c = 10, 12$ the system of equations

$$\begin{aligned} x + 2y + z &= 4 \\ 2x + y + z &= 2 \\ 4x + 5y + 3z &= c \end{aligned}$$

has no solution. Determine the value for which no solution exists. [*Hint*: $2(1,2,1) + (2,1,1) = (4,5,3)$.]

2. (*15 points*) Let V be a vector space, and suppose that $\{v_1, v_2, v_3\}$ is a linearly independent subset of V. Show that $\{v_1 + v_2, v_2, v_3\}$ is also linearly independent.

3. (*15 points*) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

4. (*20 points*) Express the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 6 & 5 \end{pmatrix}$$

as a product LU where U is upper triangular and L is lower unitriangular.

5. (*10 points*) Suppose that A is an $m \times n$ matrix, B is an $m \times 1$ matrix, and X_1 and X_2 solve the matrix equation $AX = B$. Show that their average value $\frac{1}{2}(X_1 + X_2)$ is also a solution.

6. (*10 points*) Is there a matrix A of the form

$$\begin{pmatrix} a & b & 1 & 0 & 0 \\ c & d & 0 & 1 & 0 \\ e & f & 0 & 0 & 1 \end{pmatrix}$$

whose row space is 2-dimensional? Either provide an example or explain why such a matrix cannot exist.

7. (*20 points*) Find a nonzero vector in $\mathbf{R}^4$ that is orthogonal to each of the vectors $(1, 2, 3, 6)$, $(0, 1, 2, 3)$, $(0, 0, 1, 2)$.