SOLUTIONS TO GENERAL LOGISTIC EQUATIONS

This is a sequel to `logistic-equation.*` and shows how one can take the general solution for the equation

$$y' \quad = \quad a\,y \;-\; b\,y^2 \quad = \quad r(M-y)y$$

in the form

$$y \quad = \quad \frac{Ke^{ax}}{1+cKe^{ax}}$$

(where $c = b/a$ and K is a constant of integration) and rewrite it in the alternative form described in Exercise 14 on page 519 of the text. As in the previously mentioned document, the first step is to equate the coefficients of y and y^2, obtaining the equations $rM = a$ and $r = b$. It follows that $c = b/a = 1/M$.

The second step is to factor Ke^{ax} out of the denominator and simplify the resulting expression:

$$\frac{Ke^{ax}}{1+cKe^{ax}} \quad = \quad \frac{Ke^{ax}}{Ke^{ax}(c+K^{-1}e^{ax})} \quad = \quad \frac{1}{c+K^{-1}e^{-ax}}$$

We next multiply the numerator and denominator by M and recall that $M \cdot c = 1$:

$$\frac{1}{c+K^{-1}e^{ax}} \quad = \quad \frac{M}{Mc+MK^{-1}e^{-ax}} \quad = \quad \frac{M}{1+MK^{-1}e^{-ax}}$$

Now MK^{-1} is merely a rescaled constant of integration, so we might as well replace it by the symbol L. If we do this we conclude that the solution to the original differential equation may be written in the form

$$y \quad = \quad \frac{M}{1+Le^{-ax}}$$

where L is a constant of integration.■