Isovariantly exotic symmetries of products of spheres

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In the study of group actions on manifolds, the notions of equivariant and isovariant homotopy equivalence are basically the same for smooth semifree actions which satisfy a condition called the Gap Hypothesis. We shall describe examples where these two equivalence differ when the Gap Hypothesis fails, constructing actions on a product of spheres $S^m \times S^n$ which have the following properties: (i) The actions are equivariantly homotopy equivalent to standard product actions of the trivial action on S^m with a linear action on S^n. (ii) The constructed actions are not isovariantly homotopy equivalent to the standard product actions. (iii) If q is the dimension of the fixed point set, then one has a dimension relationship $m + n \geq 2q - 3$ which is very close to the Gap Hypothesis inequality.—The proof involves suitable choices of equivariant vector bundles over spheres with trivial actions, the standard homotopy-theoretic criteria for such bundles to be equivariantly fiber homotopically trivial, and information about the homotopy groups of classical groups and equivariant function spaces of spheres with linear actions. Implications for equivariant surgery will also be discussed.