Addendum to Section 4:

Tangential self-equivalences of lens spaces

At the beginning of this paper we ~~mention~~ stated the following result without proof:

PROPOSITION 4.2. *Let M^{2n-1} be a lens space with fundamental group $\mathbb{Z}_p$. Then* ~~$n \neq 1$ (and p is an odd prime)~~ *every tangential homotopy self-equivalence of M^{2n-1} is homotopic to a diffeomorphism (in fact, an isometry).*

This turns out to be a straightforward consequence of Folkman's theorem [ ].

PROOF. Suppose that M^{2n-1} is given by a $\mathbb{Z}_p$ representation $\varphi a_1 + \dots + \varphi a_n$, where

φ

$1 \leq a_j \leq \frac{1}{2}(p-1)$, and write $M = L(a_1, \ldots, a_n)$

If $f : M^{2n-1} \to M^{2n-1}$ is a tangential homotopy self equivalence, then there is some unit $v \in \mathbb{Z}_p$ such that f_* is multiplication by v. As usual, it follows that $L(a_1, \ldots, a_n)$ is isometric to $L(va_1, \ldots, va_n)$, where we now ~~allow~~ define $\rho_a = \rho_{p-a}$ if $\frac{1}{2}(p+1) \leq a \leq p-1$, and this yields On the universal covering space level ~~we have~~ a $\mathbb{Z}_p$-equivariant map of spheres

$$\tilde{g} : S(va_1, \ldots, va_n) \longrightarrow S(a_1, \ldots, a_n)$$

~~and~~ covering a tangential homotopy ~~self~~ equivalence

$$g : L(va_1, \ldots, va_n) \longrightarrow L(a_1, \ldots, a_n).$$

If we take an equivariant joins of $\tilde{g}$ with the identity, we can construct a new tangential homotopy ~~self~~ equivalence

$$g_0 : L(va_1,\dots,va_m,1,\dots,p-1) \to L(a_1,\dots,a_m,1,\dots,p-1).$$

Given a sequence $(b_1,\dots,b_m)$ of nonzero integers mod p and some fixed nonzero integer k mod p, let $\mu[k](b_1,\dots,b_m)$ be the number of b_i's such that $b_i \equiv \pm k\ (p)$. In our preceding examples, for each k we clearly have

$$\mu[k](va_1,\dots,va_m,1,\dots,p-1) = \mu[k](va_1,\dots,va_m)+2$$

$$\mu[k](a_1,\dots,a_m,1,\dots,p-1) = \mu[k](a_1,\dots,a_m)+2.$$

We can now apply Folkman's theorem to conclude that the two quantities on the left sides of the displayed equations are equal. Therefore we also have

$$\mu[k](va_1,\dots,va_m) = \mu[k](a_1,\dots,a_m)$$

for all k. ~~Since f has degre~~ Therefore there is an isometry φ from $L(va_1, \ldots, va_n)$ to $L(a_1, \ldots, a_n)$ which induces the identity on fundamental groups. Since g is a tangential homotopy equivalence and ~~k~~ induces the identity on fundamental groups, the maps φ and g agree on fundamental groups and have equal degrees, and hence they must be homotopic. ∎