[CONTINUATION / CONCLUSION OF SECTION 6]

~~[illegible], the map~~

As at the beginning of this section, let X be a connected finite complex. The remarks in the first paragraph of this section show that, if we take the loop sum operation on $\{X, S^0\}$ and the direct sum operation on $[X, G/Top]$, then the composite

$$\{X, S^0\} \longrightarrow [X, SG] \longrightarrow [X, G/Top]$$

DISPLAY

is not usually additive. However, we have the following useful result:

PROPOSITION 6.5. <u>In the setting above, there is an infinite loop space structure on G/Top such that the displayed composite is a homomorphism.</u>

In fact, this structure ~~is the~~ structure is given by suitable versions of D Sullivan's Characteristic Variety Theorem (compare SULLIVAN 1967 NOTES [ ], JONES [ ] or NICAS [ ]).

PROOF. If X is a closed oriented manifold, then the infinite loop space structure on G/Top has the following ~~interpretation~~ description on the set $[X, G/Top]$: Take Sullivan's family of morphisms $\varphi_i : V_i \longrightarrow X$, where each V_i is either a closed manifold or a near-manifold with explicitly specified singularities, for each $\alpha \in [X, G/Top]$ construct surgery problems associated to the various classes $\varphi_i^* \alpha \in [V_i, G/Top]$, and take their Kervaire invariant or (possibly reduced) signature ~~[illegible]~~ ~~[illegible]~~ invariants which live in suitable cyclic abelian groups Λ_i. These yield an embedding of $[X, G/Top]$

into $\prod_i \Lambda_i$, and ~~then take the operation~~ the abelian group operation on $[X, G/Top]$ given by this embedding corresponds to the Characteristic Variety infinite loop space structure on G/Top (the associated spectrum is frequently denoted by symbolism like $\underset{\text{BBBOLD}}{\mathbb{L}}(1)$).

Suppose now that we are given classes u and v in $\{X, S^0\}$, and let $\chi(u), \chi(v) \in \prod \Lambda_i$ be given by the Characteristic Variety construction. We need to show that $\chi(u+v) = \chi(u) + \chi(v)$. One way of constructing tangential surgery problems associated to u and v is to begin by taking their S-duals, which lie in the stable homotopy groups $\pi_{\dim X}(X^\nu)$, where as usual ~~is~~ X^ν

~~...~~ denotes the Thom complex of the (formally) 0-dimensional stable normal bundle ν of X. If we make these dual maps "transverse to the zero section" (stably, of course), we obtain degree zero tangential normal maps (f_{i}, b_{i}) for suitable $f_{i}: Y_i \longrightarrow X$ $(i = u, v)$. The surgery problems associated to u, v, and $u+v$ are then given by

$$(f_u, b_u) \amalg id_X, \quad (f_v, b_v) \amalg id_X, \quad (f_u, b_u) \amalg (f_v, b_v) \amalg id_X$$

respectively. It is now straightforward to check that if $\chi(u)$ and $\chi(v)$ are the characteristic variety surgery obstructions for u and v respectively, then $\chi(u) + \chi(v)$ will give the characteristic variety surgery obstructions for $u+v$ (see [ ] for a more detailed analysis of such problems). ∎

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The preceding result yields a useful ~~analog generalization of Proposition 6.3 and~~ complement to Proposition 6.3. ~~and~~

PROPOSITION 6.6. *Let p be an odd prime, let X be a closed oriented manifold, and let $a \in [X, G/Top]_{(p)}$ be a class which lies in the image of*

$$[S^{2k+1}X, S^{2k+1}]_{(p)} \longrightarrow \{X, S^0\}_{(p)} \cong [X, SG]_{(p)} \longrightarrow [X, G/Top]_{(p)}$$

where $k \geq 1$. If $*$ denotes the binary operation on the codomain given by the Characteristic Variety Theorem and $*^p y$ denotes $y * y * \dots * y$ (p factors), then $*^p a$ lies in the image of $[S^{2k-1}X, S^{2k-1}]_{(p)}$.

PROOF. Let $a' \in [S^{2k+1}X, S^{2k+1}]_{(p)}$ be a preimage of a. Then if $*$ denotes the loop sum in $[S^{2k+1}X, S^{2k+1}]_{(p)}$, the results of Cohen, Moore and Neisendorfer imply that $*^p a'$ lies in the image of $[S^{2k-1}X, S^{2k-1}]_{(p)}$. Since the displayed composite is additive if we take the loop space sum on the domain and the Characteristic Variety sum on the codomain it follows that $*^p a$ lifts in the described manner. ■

In the next section we shall prove a similar result if the Characteristic Variety operation is replaced by the direct sum and X is a mod p loop space (see Proposition 7.2).