UC Riverside
MATH 007A
Summer 2022
Study Guide for the Final Examination

Questions you see here on this study guide may or may not appear on the final exam. If they do appear, then they may or may not be similar to your final exam. You are free to write all your solutions to these questions on your cheat sheet, but only if you are able to fit all or some of them into a two-sided 8x11 paper.

1. Use l'Hôpital's rule to compute the following limits. Be sure to check the indeterminate condition every time before using l'Hôpital's rule.

   (a) $\lim\limits_{x \to 5} \dfrac{x-5}{x^2-25}$

   (b) $\lim\limits_{x \to 0^+} x \ln x$

   (c) $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x^2}$

   (d) $\lim\limits_{x \to 0} \dfrac{3^x - 1}{2^x - 1}$

   (e) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$

   (f) $\lim\limits_{x \to \infty} \dfrac{x^3}{3x^3 + 2}$

   (g) $\lim\limits_{x \to 0} \dfrac{\sec x - 1}{x^2}$

   (h) $\lim\limits_{x \to 0} \dfrac{\sin x}{\cos x - 1}$

2. Prove derivatives of all trigonometric functions and inverse trigonometric functions. (e.g. prove $\dfrac{d}{dx} \sec x = \sec x \tan x$, $\dfrac{d}{dx} \tan^{-1} x = \dfrac{1}{1+x^2}$, etc.)

3. Consider the function $f(x) = \ln x$ and the interval $[1, e]$.

   (a) Find the slope of the secant line connecting the points $(1, 0)$ and $(e, 1)$.

   (b) Find the number c in the interior of $[1, e]$ such that $f'(c)$ is equal to the slope of the secant line you computed in part (a).

   (c) Find the equation of the tangent line at $x = c$, where c is the number you found in part (b).

4. Use logarithmic differentiation to find the derivative of

$$y = \frac{\sqrt{x+3}(x^2+4)^2 \ln(3x) \sin x}{(x^2-1)^{\frac{5}{3}}(x^2+3x+4)^3(x+1)^2 e^x}.$$

5. Determine where the function

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$$

is increasing and decreasing, and where it is concave up and concave down.

6. Compute the derivatives of the following involving trigonometric functions.
   (a) $f(x) = \sin^3 x + \cos^3 x$
   (b) $f(x) = \dfrac{1}{\tan(2x) - x}$
   (c) $f(x) = \sec\left(\dfrac{1}{1+x^2}\right)$
   (d) $f(x) = \sqrt{\sin(2x^2 - 1)}$

7. Compute the derivatives of the following involving exponential functions.
   (a) $f(x) = e^{x^3-1}$
   (b) $f(x) = 2^{\sqrt{1-2x^3}}$
   (c) $f(x) = e^{\sqrt[5]{2x^2+3x+1}}$
   (d) $f(x) = 5^{2x^3-x}$

8. Compute the derivatives of the following involving logarithmic functions.
   (a) $f(x) = \ln(3x+4)$
   (b) $f(x) = \ln\left(\dfrac{2x}{1+x^2}\right)$
   (c) $f(x) = (\ln(1-x^2))^3$
   (d) $f(x) = \ln(\ln(\ln x))$

9. You will be asked to prove any two of the following on the final exam:
   - Power rule for positive integer exponents, using the limit definition of the derivative and the binomial theorem
   - Power rule for all real number exponents, using logarithmic differentiation
   - Chain rule, using the limit definition of the derivative
   - Product rule, using the limit definition of the derivative
   - Quotient rule, using the limit definition of the derivative
   - Quotient rule, using the product rule and chain rule