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Name: SOLUTION ______________ Student ID: ______________

UC Riverside
MATH 007A
Summer 2022
Final Examination

Write your name and student ID number on every page. This ensures that there will be no mix-up between you and your classmates when your work is being graded.

Show all work. You are *not* required to simplify your final answers, but you must perform all required computations involving limits and/or derivatives in your solution. Finally, wherever you have shown substantial work, please $\boxed{\text{box your final answers}}$!

You are allowed to use one two-sided 8×11 cheat sheet with anything you have handwritten, provided that you have already uploaded a scanned copy of it onto Gradescope. You may also use a calculator on questions that require it. All other resources are prohibited.

Please have your photo ID on your desk for the duration of the exam. Acceptable forms of photo identification include your UCR ID card, driver license, government-issued ID, or passport. The instructor will walk around the classroom during the exam to inspect your photo ID and your cheat sheet.

This should go without saying, but please uphold your academic honesty and integrity while taking this exam. Any instance of cheating could lead to an automatic zero score on this exam or, if the violation is severe enough, a failing grade in the entire course.

You have 2 hours to complete this final exam. No time extensions will be granted except for reasons approved by the Student Disability Resource Center. *Do not start the exam until your instructor tells everyone to do so.*

If you need to use the restroom during the exam, you must ask the instructor for permission and leave your phone on your desk. You cannot make up any time you missed while outside the classroom. Do not stay outside the classroom for more than five (5) minutes at a time, and do not use the restroom more than twice during the exam; otherwise, the instructor will take your exam and dismiss you early, and you will only be graded for the work done.

In order for your exam score to count, you must sign on the dotted line below, which indicates that you have read and agreed to these policies. Submit this page on Gradescope with your exam once you are finished.

X..

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(46pts) 1. Use l'Hôpital's rule to compute the following limits. Be sure to check the indeterminate condition every time before using l'Hôpital's rule.

(6pts) (a) $\lim_{x\to 4} \frac{x^2-16}{x-4}$

$\lim_{x\to 4} \frac{x^2-16}{x-4} = \frac{4^2-16}{4-4}$

$= \frac{0}{0}$

indeterminate

$\lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} \frac{\frac{d}{dx}(x^2-16)}{\frac{d}{dx}(x-4)}$

$= \lim_{x\to 4} \frac{2x}{1}$

$= \lim_{x\to 4} 2x$

$= 2\cdot 4$

$= \boxed{8}$

(6pts) (b) $\lim_{x\to\infty} xe^{-x}$

$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x}$

$= \frac{\infty}{e^\infty}$

$= \frac{\infty}{\infty}$

indeterminate

$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}e^x}$

$= \lim_{x\to\infty} \frac{1}{e^x}$

$= \frac{1}{e^\infty}$

$= \frac{1}{\infty}$

$= \boxed{0}$

(6pts) (c) $\lim_{x\to 0} \frac{\sin^2 x}{x}$

$\lim_{x\to 0} \frac{\sin^2 x}{x} = \frac{\sin^2 0}{0}$

$= \frac{0}{0}$

indeterminate

$\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(\sin^2 x)}{\frac{d}{dx}x}$

$= \lim_{x\to 0} \frac{2\sin x\cos x}{1}$

$= \lim_{x\to 0} 2\sin x\cos x$

$= 2\sin 0\cos 0$

$= 2\cdot 0\cdot 1$

$= \boxed{0}$

(6pts) (d) $\lim_{x\to 0} \frac{e^x-1}{\pi^x-1}$

$\lim_{x\to 0} \frac{e^x-1}{\pi^x-1} = \frac{e^0-1}{\pi^0-1}$

$= \frac{1-1}{1-1}$

$= \frac{0}{0}$

indeterminate

$\lim_{x\to 0} \frac{e^x-1}{\pi^x-1} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^x-1)}{\frac{d}{dx}(\pi^x-1)}$

$= \lim_{x\to 0} \frac{e^x}{\pi^x\ln\pi}$

$= \frac{e^0}{\pi^0\ln\pi}$

$= \frac{1}{1\ln\pi}$

$= \boxed{\frac{1}{\ln\pi}}$

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Question 1 continued.

(6pts) (e) $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2} = \frac{e^0 - 0 - 1}{0^2} = \frac{1-1}{0} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^x - x - 1)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{e^x - 1 - 0}{2x} = \lim_{x\to 0} \frac{e^x - 1}{2x} = \frac{e^0 - 1}{2\cdot 0} = \frac{1-1}{0} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2} = \lim_{x\to 0} \frac{e^x - 1}{2x} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^x - 1)}{\frac{d}{dx}(2x)} = \lim_{x\to 0} \frac{e^x - 0}{2} = \lim_{x\to 0} \frac{e^x}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$$

(6pts) (f) $\lim_{x\to\infty} \frac{\sqrt{x}}{4x+9}$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{4x+9} = \frac{\sqrt{\infty}}{4\cdot\infty + 9} = \frac{\infty}{\infty}$$

indeterminate

$$\lim_{x\to\infty} \frac{\sqrt{x}}{4x+9} = \lim_{x\to\infty} \frac{\frac{d}{dx}\sqrt{x}}{\frac{d}{dx}(4x+9)} = \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{4} = \lim_{x\to\infty} \frac{1}{8\sqrt{x}} = \frac{1}{8\sqrt{\infty}} = \frac{1}{\infty} = \boxed{0}$$

(6pts) (g) $\lim_{x\to 0} \frac{\tan x}{x^2}$

$$\lim_{x\to 0} \frac{\tan x}{x^2} = \frac{\tan 0}{0^2} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{\tan x}{x^2} = \lim_{x\to 0} \frac{\frac{d}{dx}(\tan x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{\sec^2 x}{2x} = \boxed{\text{DNE}}$$

because

$$\lim_{x\to 0^-} \frac{\sec^2 x}{2x} = -\infty, \quad \lim_{x\to 0^+} \frac{\sec^2 x}{2x} = +\infty,$$

(6pts) (h) $\lim_{x\to 0} \frac{\sin(x^2)}{1 - \cos x}$

$$\lim_{x\to 0} \frac{\sin(x^2)}{1-\cos x} = \frac{\sin(0^2)}{1-\cos 0} = \frac{0}{1-1} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{\sin(x^2)}{1-\cos x} = \lim_{x\to 0} \frac{\frac{d}{dx}\sin(x^2)}{\frac{d}{dx}(1-\cos x)} = \lim_{x\to 0} \frac{\cos(x^2)\cdot 2x}{0-(-\sin x)} = \lim_{x\to 0} \frac{2x\cos(x^2)}{\sin x} = \frac{2\cdot 0\cdot\cos(0^2)}{\sin 0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin(x^2)}{1-\cos x} = \lim_{x\to 0} \frac{2x\cos(x^2)}{\sin x} = \lim_{x\to 0} \frac{\frac{d}{dx}(2x\cos(x^2))}{\frac{d}{dx}\sin x} = \lim_{x\to 0} \frac{2\cos(x^2) - 4x^2\sin(x^2)}{\cos x} = \frac{2\cos(0^2) - 4\cdot 0^2\sin(0^2)}{\cos 0} = \frac{2\cdot 1 - 0}{1} = \boxed{2}$$

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(12pts) 2. Use the limit definition of the derivative to prove the following trigonometric identities.

(6pts) (a) Use the limit definition of the derivative to prove

$$\frac{d}{dx}\sin x = \cos x$$

Hint: Use the trigonometric identity

$$\sin(x+h) = \sin x\cos h + \cos x\sin h.$$

$$\frac{\sin(x+h)-\sin x}{h} = \frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$= \frac{\sin x(\cos h - 1) + \cos x\sin h}{h}$$

$$= \sin x\frac{\cos h - 1}{h} + \cos x\frac{\sin h}{h}$$

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\left(\sin x\frac{\cos h - 1}{h} + \cos x\frac{\sin h}{h}\right)$$

$$= \sin x\lim_{h\to 0}\frac{\cos h - 1}{h} + \cos x\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \sin x\cdot 0 + \cos x\cdot 1$$

$$= \cos x$$

(6pts) (b) Use the limit definition of the derivative to prove

$$\frac{d}{dx}\cos x = -\sin x$$

Hint: Use the trigonometric identity

$$\cos(x+h) = \cos x\cos h - \sin x\sin h.$$

$$\frac{\cos(x+h)-\cos x}{h} = \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \frac{\cos x(\cos h - 1) - \sin x\sin h}{h}$$

$$= \cos x\frac{\cos h - 1}{h} - \sin x\frac{\sin h}{h}$$

$$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0}\left(\cos x\frac{\cos h - 1}{h} - \sin x\frac{\sin h}{h}\right)$$

$$= \cos x\lim_{h\to 0}\frac{\cos h - 1}{h} - \sin x\lim_{h\to 0}\frac{\sin h}{h}$$

$$= \cos x\cdot 0 - \sin x\cdot 1$$

$$= -\sin x$$

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(18pts) ~~3~~ 2. Prove the following derivatives.

(6pts) (a) Use $\tan x = \dfrac{\sin x}{\cos x}$ and the quotient rule to prove

$$\frac{d}{dx}\tan x = \sec^2 x.$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$= \frac{\left(\frac{d}{dx}\sin x\right)\cos x - \sin x \frac{d}{dx}\cos x}{(\cos x)^2}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

(6pts) (b) Use $\csc x = \dfrac{1}{\sin x}$ and the quotient rule or chain rule to prove

$$\frac{d}{dx}\csc x = -\csc x \cot x.$$

$$\frac{d}{dx}\csc x = \frac{d}{dx}\frac{1}{\sin x}$$

$$= \frac{d}{dx}(\sin x)^{-1}$$

$$= -1(\sin x)^{-2}\cdot\frac{d}{dx}\sin x$$

$$= -\frac{1}{\sin^2 x}\cos x$$

$$= -\frac{1}{\sin x}\frac{\cos x}{\sin x} = -\csc x \cot x$$

(6pts) (c) Use $\cot x = \dfrac{\cos x}{\sin x}$ and the quotient rule to prove

$$\frac{d}{dx}\cot x = -\csc^2 x.$$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\frac{\cos x}{\sin x}$$

$$= \frac{\left(\frac{d}{dx}\cos x\right)\sin x - \cos x\frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{(-\sin x)\sin x - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x}$$

$$= -\csc^2 x$$

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(18pts) 4. Compute the derivatives of the following inverse trigonometric functions.

(6pts) (a) Prove $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$.

Hint: Set $y = \sin^{-1}x$, and write it as $\sin y = x$. Then use implicit differentiation on $\sin y = x$ to compute $\frac{dy}{dx}$.

$y = \sin^{-1} x$

$\sin y = x$

$\frac{d}{dx}(\sin y) = \frac{d}{dx} x$

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{\cos^2 y}} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$

(6pts) (b) Prove $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$.

Hint: Set $y = \cos^{-1}x$, and write it as $\cos y = x$. Then use implicit differentiation on $\cos y = x$ to compute $\frac{dy}{dx}$.

$y = \cos^{-1} x$

$\cos y = x$

$\frac{d}{dx}\cos y = \frac{d}{dx} x$

$-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{\sin^2 y}} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$

(6pts) (c) Prove $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$.

Hint: Set $y = \tan^{-1}x$, and write it as $\tan y = x$. Then use implicit differentiation on $\tan y = x$ to compute $\frac{dy}{dx}$.

$y = \tan^{-1} x$

$\tan y = x$

$\frac{d}{dx}(\tan y) = \frac{d}{dx} x$

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

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(12pts) ~~4.~~ 3. Consider the function

$$f(x) = x \ln x$$

and the interval $[1, e]$.

(2pts) (a) Find the slope of the secant line connecting the points $(1, 0)$ and ~~(e, 1)~~ (e, e).

$$m = \frac{e-0}{e-1} = \boxed{\frac{e}{e-1}}$$

(5pts) (b) Find the number c in the interior of $[1, e]$ such that $f'(c)$ is equal to the slope of the secant line you computed in part (a).

$$f'(x) = \frac{d}{dx}(x \ln x) = \left(\frac{d}{dx}x\right)\ln x + x\frac{d}{dx}\ln x = 1 \cdot \ln x + x\frac{1}{x} = \ln x + 1$$

$$f'(c) = m$$

$$\ln c + 1 = \frac{e}{e-1}$$

$$\ln c = \frac{e}{e-1} - 1 = \frac{1}{e-1}$$

$$\boxed{c = e^{\frac{1}{e-1}}}$$

(5pts) (c) Find the equation of the tangent line at $x = c$, where c is the number you found in part (b).

$$c = e^{\frac{1}{e-1}}$$

$$f'(c) = m = \frac{e}{e-1}$$

$$f(c) = c \ln c = e^{\frac{1}{e-1}} \ln\left(e^{\frac{1}{e-1}}\right) = e^{\frac{1}{e-1}}\left(\frac{1}{e-1}\ln e\right) = \frac{1}{e-1}e^{\frac{1}{e-1}}$$

$$y - f(c) = f'(c)(x - c)$$

$$y - \frac{1}{e-1}e^{\frac{1}{e-1}} = \frac{e}{e-1}\left(x - e^{\frac{1}{e-1}}\right)$$

$$y - \frac{1}{e-1}e^{\frac{1}{e-1}} = \frac{e}{e-1}x - \frac{e e^{\frac{1}{e-1}}}{e-1}$$

$$y = \frac{e}{e-1}x + \frac{1}{e-1}e^{\frac{1}{e-1}} - \frac{e e^{\frac{1}{e-1}}}{e-1}$$

$$y = \frac{e}{e-1}x + \frac{e^{\frac{1}{e-1}}(1-e)}{e-1}$$

$$\boxed{y = \frac{e}{e-1}x - e^{\frac{1}{e-1}}}$$

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(25pts) 4, Use logarithmic differentiation to find the derivative of

$$y = \frac{(x^2+4)^{\frac{3}{2}}(x^3+3x^2+5x+2)e^x\cos x}{\sqrt{3x+7}(x^2+2x+2)^6(5x+3)^2\ln(2x)}.$$

$$\ln y = \ln\left(\frac{(x^2+4)^{\frac{3}{2}}(x^3+3x^2+5x+2)e^x\cos x}{\sqrt{3x+7}\,(x^2+2x+2)^6(5x+3)^2\ln(2x)}\right)$$

$$= \ln(x^2+4)^{\frac{3}{2}} + \ln(x^3+3x^2+5x+2) + \ln(e^x) + \ln(\cos x) - \ln\sqrt{3x+7} - \ln(x^2+2x+2)^6 - \ln(5x+3)^2 - \ln(\ln(2x))$$

$$= \frac{3}{2}\ln(x^2+4) + \ln(x^3+3x^2+5x+2) + x + \ln(\cos x) - \frac{1}{2}\ln(3x+7) - 6\ln(x^2+2x+2) - 2\ln(5x+3) - \ln(\ln(2x))$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\ln y = \frac{d}{dx}\left[\frac{3}{2}\ln(x^2+4) + \ln(x^3+3x^2+5x+2) + x + \ln(\cos x) - \frac{1}{2}\ln(3x+7) - 6\ln(x^2+2x+2) - 2\ln(5x+3) - \ln(\ln(2x))\right]$$

$$= \frac{3}{2}\frac{2x}{x^2+4} + \frac{3x^2+6x+5}{x^3+3x^2+5x+2} + 1 - \frac{\sin x}{\cos x} - \frac{1}{2}\frac{3}{3x+7} - 6\frac{2x+2}{x^3+3x^2+5x+2} - 2\frac{5}{5x+3} - \frac{1}{\ln(2x)}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = y\left[\frac{3}{2}\frac{2x}{x^2+4} + \frac{3x^2+6x+5}{x^3+3x^2+5x+2} + 1 - \frac{\sin x}{\cos x} - \frac{1}{2}\frac{3}{3x+7} - 6\frac{2x+2}{x^3+3x^2+5x+2} - 2\frac{5}{5x+3} - \frac{1}{x\ln(2x)}\right]$$

$$= \boxed{\frac{(x^2+4)^{\frac{3}{2}}(x^3+3x^2+5x+2)e^x\cos x}{\sqrt{3x+7}\,(x^2+2x+2)^6(5x+3)^2\ln(2x)}\left[\frac{3}{2}\frac{2x}{x^2+4} + \frac{3x^2+6x+5}{x^3+3x^2+5x+2} + 1 - \frac{\sin x}{\cos x} - \frac{1}{2}\frac{3}{3x+7} - 6\frac{2x+2}{x^3+3x^2+5x+2} - 2\frac{5}{5x+3} - \frac{1}{x\ln(2x)}\right]}$$

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(24pts) ~~4.~~ 5. Compute the derivatives of the following involving trigonometric functions.

(6pts) (a) $f(x) = \dfrac{\sec(x^2-1)}{2-x^2}$

$$f'(x) = \frac{d}{dx}\left(\frac{\sec(x^2-1)}{2-x^2}\right)$$

$$= \frac{\left(\frac{d}{dx}\sec(x^2-1)\right)(2-x^2) - \sec(x^2-1)\frac{d}{dx}(2-x^2)}{(2-x^2)^2}$$

$$= \boxed{\frac{[\sec(x^2-1)\tan(x^2-1)\cdot 2x](2-x^2) - \sec(x^2-1)(-2x)}{(2-x^2)^2}}$$

(6pts) (b) $f(x) = \tan(\sin(\cos(3x)))$

$$f'(x) = \frac{d}{dx}\tan(\sin(\cos(3x)))$$

$$= \sec^2(\sin(\cos(3x)))\frac{d}{dx}(\sin(\cos(3x)))$$

$$= \sec^2(\sin(\cos(3x)))\cos(\cos(3x))\frac{d}{dx}\cos(3x)$$

$$= \boxed{\sec^2(\sin(\cos(3x)))\cos(\cos(3x))[-\sin(3x)\cdot 3]}$$

(6pts) (c) $f(x) = \dfrac{\csc(x^2)}{\csc^2 x}$

$$f'(x) = \frac{d}{dx}\left(\frac{\csc(x^2)}{\csc^2 x}\right)$$

$$= \frac{\left(\frac{d}{dx}\csc(x^2)\right)\csc^2 x - \csc(x^2)\frac{d}{dx}\csc^2 x}{(\csc^2 x)^2}$$

$$= \boxed{\frac{[-\csc(x^2)\cot(x^2)\cdot 2x]\csc^2 x - \csc(x^2)[2\csc x\cdot(-\csc x\cot x)]}{\csc^4 x}}$$

(6pts) (d) $f(x) = \cot\left(\dfrac{1}{x}\right)$

$$f'(x) = \frac{d}{dx}\cot\left(\frac{1}{x}\right)$$

$$= -\csc^2\left(\frac{1}{x}\right)\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \boxed{-\csc^2\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)}$$

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(24pts) ~~4.~~ 6, Compute the derivatives of the following involving exponential functions.

(6pts) (a) $f(x) = e^{x^2 - \frac{1}{x^2}}$

$$f'(x) = \frac{d}{dx}\left(e^{x^2-\frac{1}{x^2}}\right)$$
$$= e^{x^2-\frac{1}{x^2}} \cdot \frac{d}{dx}\left(x^2 - \frac{1}{x^2}\right)$$
$$= \boxed{e^{x^2-\frac{1}{x^2}} \cdot \left(2x + \frac{2}{x^3}\right)}$$

(6pts) (b) $f(x) = 3^{\sqrt{1-5x^3}}$

$$f'(x) = \frac{d}{dx} 3^{\sqrt{1-5x^3}}$$
$$= 3^{\sqrt{1-5x^3}} \ln 3 \frac{d}{dx}\sqrt{1-5x^3}$$
$$= 3^{\sqrt{1-5x^3}} \ln 3 \frac{1}{2\sqrt{1-5x^3}} \frac{d}{dx}(1-5x^3)$$
$$= \boxed{3^{\sqrt{1-5x^3}} \ln 3 \frac{1}{2\sqrt{1-5x^3}} \cdot (-15x^2)}$$

(6pts) (c) $f(x) = 4^{\sqrt[3]{3x^4-5}}$

$$f'(x) = \frac{d}{dx}\left(4^{\sqrt[3]{3x^4-5}}\right)$$
$$= 4^{\sqrt[3]{3x^4-5}} \ln 4 \frac{d}{dx}\sqrt[3]{3x^4-5}$$
$$= 4^{\sqrt[3]{3x^4-5}} \ln 4 \left[\frac{1}{3}(3x^4-5)^{-\frac{2}{3}} \frac{d}{dx}(3x^4-5)\right]$$
$$= \boxed{4^{\sqrt[3]{3x^4-5}} \ln 4 \left[\frac{1}{3}(3x^4-5)^{-\frac{2}{3}} \cdot 12x^3\right]}$$

(6pts) (d) $f(x) = 2^{(x^2+3x)^9}$

$$f'(x) = \frac{d}{dx}\left(2^{(x^2+3x)^9}\right)$$
$$= 2^{(x^2+3x)^9} \ln 2 \frac{d}{dx}(x^2+3x)^9$$
$$= 2^{(x^2+3x)^9} \ln 2 \left[9(x^2+3x)^8 \frac{d}{dx}(x^2+3x)\right]$$
$$= \boxed{2^{(x^2+3x)^9} \ln 2 \left[9(x^2+3x)^8 (2x+3)\right]}$$

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(24pts) 7. Compute the derivatives of the following involving logarithmic functions.

(6pts) (a) $f(x) = \ln(2x^4 + 1)$

$$f'(x) = \frac{d}{dx}\ln(2x^4+1)$$
$$= \frac{1}{2x^4+1}\frac{d}{dx}(2x^4+1)$$
$$= \boxed{\frac{1}{2x^4+1}\cdot 8x^3}$$

(6pts) (b) $f(x) = \ln\left(\frac{x}{x-1}\right)$

$$f'(x) = \frac{d}{dx}\ln\left(\frac{x}{x-1}\right)$$
$$= \frac{1}{\frac{x}{x-1}}\frac{d}{dx}\left(\frac{x}{x-1}\right)$$
$$= \frac{x-1}{x}\frac{\left(\frac{d}{dx}x\right)(x-1) - x\frac{d}{dx}(x-1)}{(x-1)^2}$$
$$= \boxed{\frac{x-1}{x}\frac{1(x-1) - x\cdot 1}{(x-1)^2}} = \frac{x-1}{x}\frac{-1}{(x-1)^2} = \boxed{-\frac{1}{x(x-1)}}$$

(6pts) (c) $f(x) = (\ln(x^2 - 2))^4$

$$f'(x) = \frac{d}{dx}(\ln(x^2-2))^4$$
$$= 4(\ln(x^2-2))^3\frac{d}{dx}\ln(x^2-2)$$
$$= 4(\ln(x^2-2))^3\frac{1}{x^2-2}\frac{d}{dx}(x^2-2)$$
$$= \boxed{4(\ln(x^2-2))^3\frac{1}{x^2-2}2x}$$

(6pts) (d) $f(x) = \ln\left(\frac{x^4+1}{x^2+1}\right)$

$$f'(x) = \frac{d}{dx}\ln\left(\frac{x^4+1}{x^2+1}\right)$$
$$= \frac{1}{\frac{x^4+1}{x^2+1}}\frac{d}{dx}\left(\frac{x^4+1}{x^2+1}\right)$$
$$= \frac{x^2+1}{x^4+1}\frac{\frac{d}{dx}(x^4+1)(x^2+1) - (x^4+1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$
$$= \boxed{\frac{x^2+1}{x^4+1}\frac{4x^3(x^2+1) - (x^4+1)(2x)}{(x^2+1)^2}}$$

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(35pts) ~~10.~~ 8. Let $f(x)$ and $g(x)$ be differentiable functions with $g(x) \neq 0$. Prove the quotient rule

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

in the following ways.

(20pts) (a) Use the limit definition of the derivative to prove the quotient rule.

See the lecture notes of our meeting on 6/30.

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Question ~~10~~ 8 continued

(15pts) (b) Use the product rule

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

and the chain rule

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

to prove the quotient rule. (Do not prove the product rule or the chain rule here.)

See the lecture notes of our meeting on 6/30.