Finishing up Section 3.2

$\lim_{x \to 3} x^2 = \lim_{x \to 3} (x \cdot x) = \lim_{x \to 3} x \cdot \lim_{x \to 3} x = 3 \cdot 3 = 9$

Example: $\lim_{x \to 3} \sin\left(\frac{\pi(x^2-1)}{4}\right)$

$\sin(x)$ is continuous on $\mathbb{R}$

$$\lim_{x \to 3} \sin\left(\frac{\pi(x^2-1)}{4}\right) = \sin\left(\lim_{x \to 3} \frac{\pi(x^2-1)}{4}\right)$$

$$= \sin\left(\frac{\lim_{x \to 3} \pi(x^2-1)}{\lim_{x \to 3} 4}\right)$$

$$= \sin\left(\frac{\pi \lim_{x \to 3} (x^2-1)}{\lim_{x \to 3} 4}\right)$$

$$= \sin\left(\frac{\pi\left(\lim_{x \to 3} x^2 - \lim_{x \to 3} 1\right)}{\lim_{x \to 3} 4}\right)$$

$$= \sin\left(\frac{\pi(9-1)}{4}\right)$$

$$= \sin(\pi \cdot 2)$$

$$= \sin(2\pi)$$

$$= \boxed{0}$$

Fewer steps

$$\lim_{x \to 3} \sin\left(\frac{\pi(x^2-1)}{4}\right) = \sin\left(\frac{\pi(3^2-1)}{4}\right)$$

$$= \sin(2\pi)$$

$$= \boxed{0}$$

Example $\lim_{x\to 2} \sqrt{2x^2-8}$ [illegible]

[illegible]

$\lim \sqrt{2x^2-8} = \sqrt{\lim (2x^2-8)}$ [illegible]

$= \sqrt{\lim 2x^2 - \lim 8}$ [illegible]

[illegible]

<u>Fewer steps</u>

$\lim \sqrt{2x^2-2} = \sqrt{2(\ldots)^2-2}$ [illegible]

[illegible]

Example $\lim_{x\to 0} e^{x-1}$

$\lim_{x\to 0} e^{x-1} = e^{\lim_{x\to 0}(x-1)}$

$= e^{\lim x - \lim 1}$

$= e^{0-1}$

$= \boxed{e^{-1}}$

$= \boxed{\frac{1}{e}}$

<u>Fewer steps</u>

$\lim_{x\to 0} e^{x-1} = e^{0-1}$

$= \boxed{e^{-1}}$ or $\boxed{\frac{1}{e}}$

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Example

$$\lim_{x \to 0} \frac{\sqrt{2x^2+25}-5}{x^2}$$

$$\lim_{x \to 0} \frac{\sqrt{2x^2+25}-5}{x^2} = \lim_{x \to 0} \frac{\sqrt{2x^2+25}-5}{x^2} \cdot \frac{\sqrt{2x^2+25}+5}{\sqrt{2x^2+25}+5}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{2x^2+25}\right)^2 - 5^2}{x^2\left(\sqrt{2x^2+25}+5\right)}$$

$$= \lim_{x \to 0} \frac{2x^2 + \cancel{25 - 25}}{x^2\left(\sqrt{2x^2+25}+5\right)}$$

$$= \lim_{x \to 0} \frac{2x^2}{x^2\left(\sqrt{2x^2+25}+5\right)}$$

$$= \lim_{x \to 0} \frac{2}{\sqrt{2x^2+25}+5}$$

Difference of squares

$$(a+b)(a-b) = a^2 - b^2$$

Here, $a = \sqrt{2x^2+25}$

$b = 5$

$$= \frac{2}{\sqrt{2 \cdot 0^2+25}+5}$$

$$= \frac{2}{\sqrt{25}+5}$$

$$= \frac{2}{5+5}$$

$$= \frac{2}{10} = \boxed{\frac{1}{5}}$$

Example: $\lim_{x \to \frac{\pi}{3}} \sin\left(\frac{x}{2}\right)$.

$$\lim_{x \to \frac{\pi}{3}} \sin\left(\frac{x}{2}\right) = \sin\left(\frac{\frac{\pi}{3}}{2}\right)$$

$$\frac{\frac{\pi}{3}}{2} = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$$

$$= \sin\left(\frac{\pi}{6}\right)$$

$$= \boxed{\frac{1}{2}}$$

Example: $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}\cdot\frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3} = \lim_{x\to 0}\frac{x^2+\cancel{9}-\cancel{9}}{x^2\left(\sqrt{x^2+9}+3\right)}$$

$$= \lim_{x\to 0}\frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt{x^2+9}+3\right)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{x^2+9}+3}$$

$$= \frac{1}{\sqrt{0^2+9}+3}$$

$$= \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

Example: $\lim_{x\to -\frac{\pi}{2}} \cos(2x)$

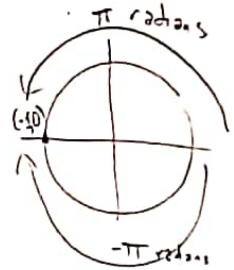
$$\lim_{x\to -\frac{\pi}{2}} \cos(2x) = \cos\left(2\left(-\frac{\pi}{2}\right)\right)$$

$$= \cos(-\pi)$$

$$= \boxed{-1}$$

Also,

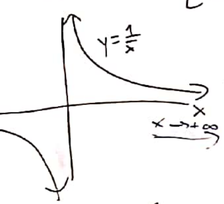
$$\lim_{x\to \frac{\pi}{2}} \cos(2x) = \cos(\pi) = -1$$



$\cos\pi = -1$

Section 3.3 Limits at Infinity

Example: $\lim_{x\to\infty} \frac{x}{x+1}$



$$\lim_{x\to\infty} \frac{x}{x+1} = \frac{\infty}{\infty+1} = \frac{\infty}{\infty} = \text{indeterminate}$$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

Instead, do this:

$$\lim_{x\to\infty} \frac{x}{x+1} = \lim_{x\to\infty} \frac{x}{x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{x\cdot\frac{1}{x}}{x\cdot\frac{1}{x}+1\cdot\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}}$$

$$= \frac{\lim_{x\to\infty} 1}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x}}$$

$$= \frac{1}{1+0}$$

$$= \boxed{1}$$

"Arithmetic" with ∞

$$\infty + \infty = \infty$$
$$\infty - \infty = \text{indeterminate}$$
$$\infty \cdot \infty = \infty$$
$$\frac{\infty}{\infty} = \text{indeterminate}$$

$$\infty + 3 = \infty$$
$$\infty - 3 = \infty$$
$$3 \cdot \infty = \infty$$
$$\frac{\infty}{3} = \infty$$
$$\frac{\infty}{0} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = \infty$$

Example: $\lim_{x\to\infty} \frac{2x^3-4x+7}{3x^3+7x^2-1}$

$$\lim_{x\to\infty} \frac{2x^3-4x+7}{3x^3+7x^2-1} = \lim_{x\to\infty} \frac{2x^3-4x+7}{3x^3+7x^2-1}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{2-\frac{4}{x^2}+\frac{7}{x^3}}{3+\frac{7}{x}-\frac{1}{x^3}}$$

$$= \frac{\lim_{x\to\infty}\left(2-\frac{4}{x^2}+\frac{7}{x^3}\right)}{\lim_{x\to\infty}\left(3+\frac{7}{x}-\frac{1}{x^3}\right)}$$

$$= \frac{2-0+0}{3+0-0}$$

$$= \boxed{\frac{2}{3}}$$

Examp

Example: $\lim\limits_{x\to\infty} \dfrac{x^2+2x-1}{x^3-3x+1}$

$$\lim_{x\to\infty}\frac{x^2+2x-1}{x^3-3x+1} = \lim_{x\to\infty}\frac{\dfrac{x^2+2x-1}{x^3-3x+1}\cdot\dfrac{1}{x^3}}{\dfrac{1}{x^3}}$$

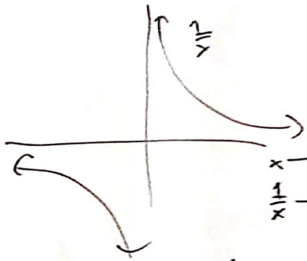
$$= \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}-\frac{1}{x^3}}{1-\frac{3}{x^2}+\frac{1}{x^3}}$$

$$= \frac{0+0-0}{1-0+0}$$

$$= \boxed{0}$$

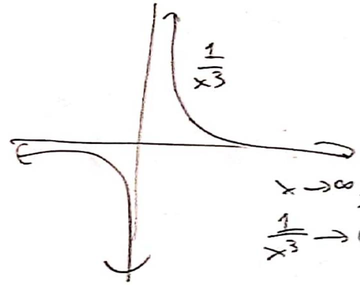
If the degrees of leading terms are DIFFERENT, multiply and divide by

$$\frac{1}{\text{Leading term with HIGHER DEGREE}}$$



$x \to \infty$

$\frac{1}{x} \to 0$



$x \to \infty$,

$\frac{1}{x^3} \to 0$

x	$\frac{1}{x}$
100	$\frac{1}{100}$
1000	$\frac{1}{1000}$
⋮	⋮
very large	very small

$x \to \infty$ $\quad \frac{1}{x} \to 0$

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Example: $\lim_{x\to\infty} \frac{x^4+2x-5}{x^2-x+2}$

$$\lim_{x\to\infty} \frac{x^4+2x-5}{x^2-x+2} = \lim_{x\to\infty} \frac{x^4+2x-5}{x^2-x+2} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{2}{x^3}-\frac{5}{x^4}}{\frac{1}{x^2}-\frac{1}{x^3}+\frac{2}{x^4}}$$

$$= \frac{1+0-0}{0-0+0}$$

$$= \frac{1}{0}$$

$$= \boxed{\infty}$$

LEADING TERMS :

$$\lim_{x\to\infty} \frac{x^4+2x-5}{x^2-x+2} = \lim_{x\to\infty} \frac{x^4}{x^2}$$

$$= \lim_{x\to\infty} x^2$$

$$= \infty$$

$$= \boxed{\infty}$$

Example: $\lim_{x\to-\infty} \frac{1-x+2x^2}{3x-5x^2}$

TAKE THE LEADING TERMS

$$\lim_{x\to-\infty} \frac{1-x+2x^2}{3x-5x^2} = \lim_{x\to-\infty} \frac{2x^2}{-5x^2}$$

$$= \lim_{x\to-\infty} \frac{2}{-5}$$

$$= \boxed{-\frac{2}{5}}$$

Example: $\lim\limits_{x\to\infty} \dfrac{1-x^3}{1+x^5}$

TAKE THE LEADING TERMS:

$$\lim_{x\to\infty} \frac{1-x^3}{1+x^5} = \lim_{x\to\infty} \frac{-x^3}{x^5}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{x^3}{x^5} = x^{3-5} = x^{-2} = \frac{1}{x^2}$$

$$= \lim_{x\to\infty} \frac{-1}{x^2}$$

$$= -\lim_{x\to\infty} \frac{1}{x^2}$$

Limit Laws explanation

$$\lim_{x\to\infty} \frac{1}{x^2} = \lim_{x\to\infty} \left(\frac{1}{x}\cdot\frac{1}{x}\right)$$

$$= \lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} \frac{1}{x}$$

$$= 0 \cdot 0$$

$$= 0$$

$$= -0$$

$$= \boxed{0}$$

Example: $\lim\limits_{x\to\infty} \dfrac{2-x^2}{1+2x}$

$$\lim_{x\to\infty} \frac{2-x^2}{1+2x} = \lim_{x\to\infty} \frac{-x^2}{2x}$$

$$= \lim_{x\to\infty} \frac{-x}{2}$$

$$= -\frac{1}{2} \lim_{x\to\infty} x$$

$$= -\frac{1}{2} \cdot \infty$$

$$= \boxed{-\infty}$$

Mini-Quiz 6/22

Compute

$$\lim_{x \to \infty} \frac{2x^5 + 7}{x^6 + 3x^3 + 1}$$

(Use any method that works for you.)