Mini-Quiz 7/14 SOLUTION

Let $f(x) = 3x^2 + 2x - 3$.

Find $\left.\frac{d}{dx} f^{-1}(x)\right|_{x=2}$.

(1) Find $f^{-1}(2)$

Set $f(x) = 2$

$$3x^2 + 2x - 3 = 2$$

Guess: $x = 1$

$$3 \cdot 1^2 + 2 \cdot 1 - 3 = 2$$
$$3 + 2 - 3 = 2$$
$$2 = 2$$

$$f(1) = 2$$
$$f^{-1}(f(1)) = f^{-1}(2)$$
$$1 = f^{-1}(2)$$

$$f^{-1}(2) = 1$$

(2) Fi

$f'(x$

② Find $f'(x)$

$$f'(x) = \frac{d}{dx}(3x^2+2x-3)$$
$$= 6x+2$$

③ Use $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$$\frac{d}{dx} f^{-1}(x)\Big|_{x=2} = \frac{1}{f'(f^{-1}(2))}$$
$$= \frac{1}{f'(1)}$$
$$= \boxed{\frac{1}{8}}$$

$$f'(x) = 6x+2$$
$$f'(1) = 6\cdot 1 + 2$$
$$= 8$$

5.5 l'Hôpital's Rule

l'Hôpital's Rule Suppose that f and g are differentiable functions and

$$\boxed{\begin{array}{l} \lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0 \qquad \lim_{x\to c} \frac{f(x)}{g(x)} = \frac{0}{0} \\ \text{or} \\ \lim_{x\to c} f(x) = \lim_{x\to c} g(x) = \infty. \qquad \lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\infty}{\infty} \end{array}}$$

Must satisfy ONE of these indeterminate conditions

Then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}.$$

Example

SOLUTION:

lim

Scanné avec CamScanner

Example: $\lim_{x\to 3} \frac{x^2-9}{x-3}$

SOLUTION: Check indeterminate condition:

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{9-9}{3-3} = \frac{0}{0}$$

Use l'Hôpital's rule

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{\frac{d}{dx}(x^2-9)}{\frac{d}{dx}(x-3)} = \lim_{x\to 3} \frac{2x}{1} = \lim_{x\to 3} 2x = 2\cdot 3 = \boxed{6}$$

Example: $\lim\limits_{x\to\infty} \dfrac{x}{1+x}$

SOLUTION: $\lim\limits_{x\to\infty} \dfrac{x}{1+x} = \dfrac{\infty}{1+\infty}$

$= \dfrac{\infty}{\infty}$

indeterminate

$$\lim_{x\to\infty} \frac{x}{1+x} = \lim_{x\to\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}(1+x)}$$

$$= \lim_{x\to\infty} \frac{1}{1}$$

$$= \lim_{x\to\infty} 1$$

$$= \boxed{1}$$

Example: $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x}$

SOLUTION:

$\lim\limits_{x\to 0} \dfrac{e^x-1}{x} = \dfrac{e^0-1}{0}$

$= \dfrac{1-1}{0}$

$= \dfrac{0}{0}$

indeterminate

$$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^x-1)}{\frac{d}{dx}x}$$

$$= \lim_{x\to 0} \frac{e^x}{1}$$

$$= \lim_{x\to 0} e^x$$

$$= e^0$$

$$= \boxed{1}$$

Example: Evaluate

SOLUTION: $\lim\limits_{x\to 2} \dfrac{x^6-64}{x^2-4} = \dfrac{2^6-6}{2^2-}$

$= \dfrac{64-}{4-}$

$= \dfrac{0}{0}$

Example: Evaluate $\lim\limits_{x\to 2} \dfrac{x^6-64}{x^2-4}$

SOLUTION:

$$\lim_{x\to 2} \frac{x^6-64}{x^2-4} = \frac{2^6-64}{2^2-4} = \frac{64-64}{4-4} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{x^6-64}{x^2-4} = \lim_{x\to 2} \frac{\frac{d}{dx}(x^6-64)}{\frac{d}{dx}(x^2-4)} = \lim_{x\to 2} \frac{6x^5}{2x} = \lim_{x\to 2} 3x^4 = 3\cdot 2^4 = 3\cdot 16 = \boxed{48}$$

Example: Evaluate $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$

SOLUTION:

$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}\sin x}{\frac{d}{dx} x} = \lim_{x\to 0} \frac{\cos x}{1} = \lim_{x\to 0} \cos x = \cos 0 = \boxed{1}$$

Example: Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$

SOLUTION:

$$\lim_{x\to 0} \frac{1-\cos x}{x} = \frac{1-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$$

indeterminate

$$\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx} x} = \lim_{x\to 0} \frac{0-(-\sin x)}{1} = \lim_{x\to 0} \sin x = \sin 0 = \boxed{0}$$

Example: Evaluate $\lim\limits_{x\to\infty} \frac{\ln x}{x}$

SOLUTION: $\lim\limits_{x\to\infty} \frac{\ln x}{x} = \frac{\ln \infty}{\infty}$

$= \frac{\infty}{\infty}$

indeterminate

$\lim\limits_{x\to\infty} \frac{\ln x}{x} = \lim\limits_{x\to\infty} \frac{\frac{d}{dx}\ln x}{\frac{d}{dx}x}$

$= \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{1}$

$= \lim\limits_{x\to\infty} \frac{1}{x}$

$= \boxed{0}$

Example: Evaluate $\lim\limits_{x\to\infty} \frac{x^3-3x+1}{3x^3-2x^2}$

SOLUTION: $\lim\limits_{x\to\infty} \frac{x^3-3x+1}{3x^3-2x^2}$ take leading terms $= \lim\limits_{x\to\infty} \frac{x^3}{3x^3}$

$= \frac{\infty^3}{3\cdot\infty^3}$

$= \frac{\infty}{\infty}$

indeterminate

$\lim\limits_{x\to\infty} \frac{x^3-3x+1}{3x^3-2x^2} = \lim\limits_{x\to\infty} \frac{\frac{d}{dx}(x^3-3x+1)}{\frac{d}{dx}(3x^3-2x^2)}$

$= \lim\limits_{x\to\infty} \frac{3x^2-3}{9x^2-4x}$

take leading terms $= \lim\limits_{x\to\infty} \frac{3x^2}{9x^2}$

$= \lim\limits_{x\to\infty} \frac{3}{9} = \lim\limits_{x\to\infty} \frac{1}{3} = \boxed{\frac{1}{3}}$

Example: Evaluate $\lim\limits_{x\to 0}$

SOLUTION: $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1-\cos 0}{0^2}$

$= \frac{1-1}{0}$

$= \frac{0}{0}$

indeterminate

$\frac{1}{3} = \boxed{\frac{1}{3}}$

Example: Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

SOLUTION:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1-\cos 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$$

indeterminate

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} \overset{L.H.}{=} \lim_{x \to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x^2)} = \lim_{x \to 0} \frac{\sin x}{2x} = \frac{\sin 0}{2 \cdot 0} = \frac{0}{0}$$

indeterminate

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{\sin x}{2x} \overset{L.H.}{=} \lim_{x \to 0} \frac{\frac{d}{dx}\sin x}{\frac{d}{dx}2x} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \boxed{\frac{1}{2}}$$

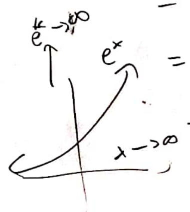
Example: Evaluate $\lim\limits_{x\to\infty} \frac{e^x}{x}$

SOLUTION:

$$\lim_{x\to\infty} \frac{e^x}{x} = \frac{e^\infty}{\infty} = \frac{\infty}{\infty}$$

indeterminate

$$\lim_{x\to\infty} \frac{e^x}{x} \overset{L.H.}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}e^x}{\frac{d}{dx}x} = \lim_{x\to\infty} \frac{e^x}{1} = \lim_{x\to\infty} e^x = \boxed{\infty}$$



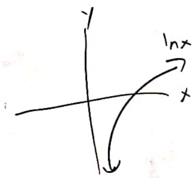
Example: Evaluate $\lim\limits_{x\to 0^+} x\ln x$

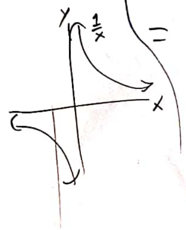
SOLUTION:

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \frac{\lim_{x\to 0^+} \ln x}{\lim_{x\to 0^+} \frac{1}{x}} = \frac{-\infty}{+\infty} = -\frac{\infty}{\infty}$$

indeterminate

Note: $\frac{\ln x}{\frac{1}{x}} = \ln x \cdot \frac{x}{1} = x\ln x$





$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$$

$$\overset{L.H.}{=} \lim_{x\to 0^+} \frac{\frac{d}{dx}\ln x}{\frac{d}{dx}\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{1}{x}\cdot(-x^2)$$

$$= \lim_{x\to 0^+} (-x)$$

$$= -0$$

$$= \boxed{0}$$

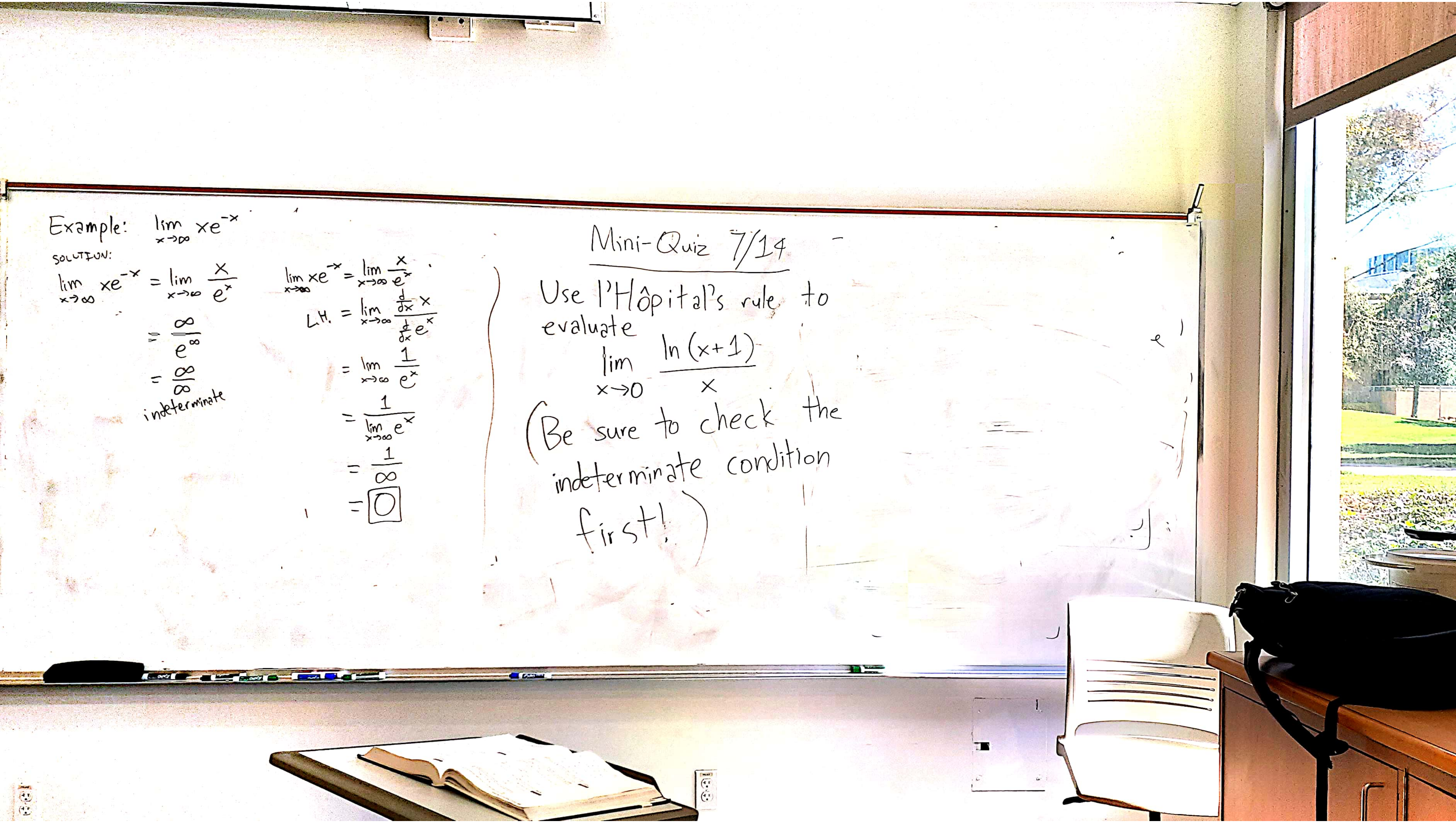
Example: $\lim\limits_{x\to\infty} xe^{-x}$

SOLUTION:

$$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{e^\infty} = \frac{\infty}{\infty}$$

indeterminate

lim


Example: $\lim_{x\to\infty} xe^{-x}$
SOLUTION:
$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{e^\infty} = \frac{\infty}{\infty}$
indeterminate
$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x}$
L.H. $= \lim_{x\to\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{\lim_{x\to\infty} e^x} = \frac{1}{\infty} = \boxed{0}$
Mini-Quiz 7/14
Use l'Hôpital's rule to evaluate
$\lim_{x\to 0} \frac{\ln(x+1)}{x}$
(Be sure to check the indeterminate condition first!)