Final Exam Saturday 7/23 8-10AM

Sproul 1340

10:05

8:00 AM James will pass out final exams

8:00 - 8:05 - Write your name and Student I.D. on EVERY page

- Also have photo ID out

8:05 - 10:05 Take the Final Exam

- You may use your 8x11 handwritten cheat sheet if you uploaded it onto Gradescope prior to the final

10:05-10:25 Stop working. Pens & pencils down.

Upload your exam solutions onto Gradescope.

You may keep your hard copy when you leave.

5.1 Continued

Theorem Mean Value Theorem

If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists at least one number $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Example: Let $f(x)=x^2$, and $0 \le x \le 1$.

(a) Find the slope of the secant line connecting the points $(0,0)$ and $(1,1)$.

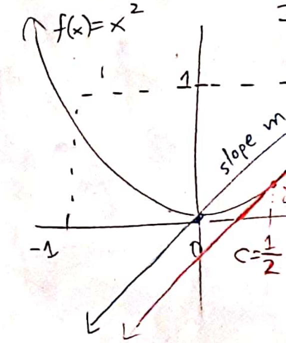
(b) Find a number c in $(0,1)$ such that $f'(c)$ is equal to the slope of the secant line you computed in part (a). Explain why such a number must exist in $(0,1)$.

(c) Find the equation of the tangent line at $x=c$.

SOLUTION: (a) $m=$

$f(1)=1^2=1$

$f(0)=0^2=0$



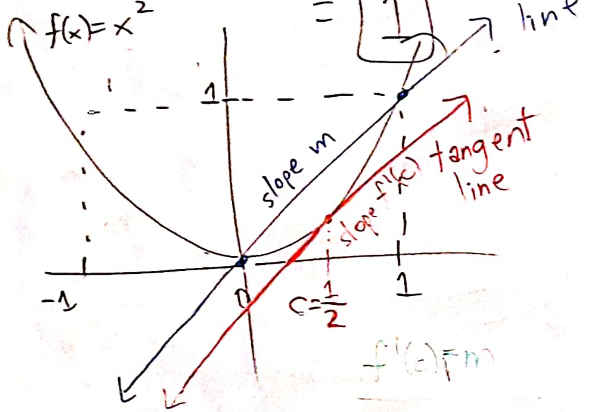
SOLUTION: (a) $m = \dfrac{f(1) - f(0)}{1 - 0}$

$f(1) = 1^2 = 1$

$= \dfrac{1 - 0}{1 - 0}$

$f(0) = 0^2 = 0$

$= \boxed{1}$



(b) $f'(x) = \frac{d}{dx}x^2 = 2x$

Set $f'(c) = m$

$2c = 1$

$c = \boxed{\frac{1}{2}}$

This number c exists by the Mean Value Theorem, since $f(x) = x^2$ is continuous on $[0,1]$ and differentiable on $(0,1)$.

(c) $c = \frac{1}{2}$

$f'(c) = m = 1$

$f(c) = f\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

Equation of the tangent line

$y - y_1 = m(x - x_1)$

$y - f(c) = f'(c)(x - c)$

$y - \frac{1}{4} = 1\left(x - \frac{1}{2}\right)$

$y - \frac{1}{4} = x - \frac{1}{2}$

$\boxed{y = x - \frac{1}{4}}$

5.1 Continued

Theorem

Mean Value Theorem

If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists at least one number $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Example: Let $f(x)=e^x$, and $0 \le x \le 1$.

(a) Find the slope of the secant line connecting the points $(0,1)$ and $(1,e)$.

(b) Find a number c in $(0,1)$ such that $f'(c)$ is equal to the slope of the secant line you computed in part (a). Explain why such a number must exist in $(0,1)$.

(c) Find the equation of the tangent line at $x=c$.

SOLUTION: (a) m

$f(1) = e^1 = e$

$f(0) = e^0 = 1$

SOLUTION: (a)

$$m = \frac{f(1)-f(0)}{1-0}$$

$f(1) = e^1 = e$

$f(0) = e^0 = 1$

$$= \frac{e-1}{1-0}$$

$$= \boxed{e-1}$$

(b)

$$f'(x) = \frac{d}{dx}e^x = e^x$$

Set $f'(c) = m$

$$e^c = e-1$$

$$\ln(e^c) = \ln(e-1)$$

$$c = \boxed{\ln(e-1)}$$

This number c exists in $[0,1]$ by the Mean Value Theorem, since $f(x)=e^x$ is continuous on $[0,1]$ and differentiable on $(0,1)$.

$$\boxed{y = (e-1)x + (e-1)[1-\ln(e-1)]}$$

(c)

$$c = \ln(e-1)$$

$$f'(c) = m = e-1$$

$$f(c) = e^c = e^{\ln(e-1)} = e-1$$

Equation of the tangent line

$$y - y_1 = m(x-x_1)$$

$$y - f(c) = f'(c)(x-c)$$

$$y-(e-1) = (e-1)(x-\ln(e-1))$$

$$y-(e-1) = (e-1)x - (e-1)\ln(e-1)$$

$$y = (e-1)x + (e-1) - (e-1)\ln(e-1)$$

$$\boxed{y = (e-1)x + (e-1)[1-\ln(e-1)]}$$

5.2 Monotonicity and Concavity

Definition A function f on the interval $[a,b]$ is called increasing on $[a,b]$ if

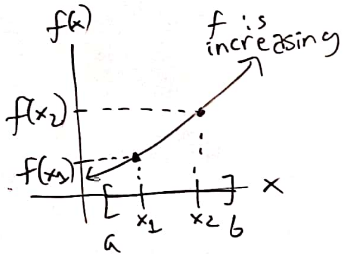
$$f(x_1) < f(x_2)$$

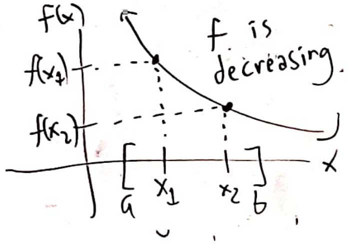
for any $x_1 < x_2$ in $[a,b]$

and is called decreasing on $[a,b]$ if

$$f(x_1) > f(x_2)$$

for any $x_1 > x_2$ in $[a,b]$.





First Derivative Test for Monotonicity

Suppose f is continuous on $[a,b]$ and differentiable on (a,b).

- If $f'(x) > 0$ for all x in (a,b), f is increasing on (a,b).
- If $f'(x) < 0$ for all x in (a,b), f is decreasing on (a,b).

Example: Determine for all real numbers x where the function

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x + 3$$

is increasing and where it is decreasing.

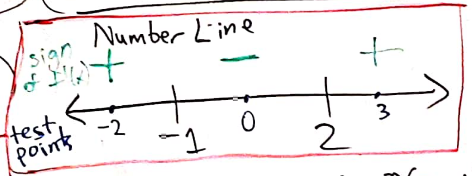
SOLUTION:

$$f'(x) = \frac{d}{dx}\left(x^3 - \frac{3}{2}x^2 - 6x + 3\right)$$
$$= 3x^2 - 3x - 6$$
$$= 3(x^2 - x - 2)$$
$$= 3(x-2)(x+1)$$

Set $f'(x) = 0$

$$3(x-2)(x+1) = 0$$
$$x - 2 = 0, \quad x + 1 = 0$$

Critical Points $x = 2, \quad x = -1$



$$f'(-2) = 3(-2-2)(-2+1) = 3\cdot(-4)\cdot(-1) = 12 > 0 \quad +$$

$$f'(0) = 3(0-2)(0+1) = 3\cdot(-2)\cdot 1 = -6 < 0 \quad -$$

$$f'(3) = 3(3-2)(3+1) = 3\cdot 1\cdot 4 = 12 > 0 \quad +$$

According to our number line,

$f(x)$ is INCREASING on $(-\infty, -1) \cup (2, \infty)$
and $f(x)$ is DECREASING on $(-1, 2)$

First Derivative Test for Monotonicity

Suppose f is continuous on $[a, b]$ and differentiable on (a, b).

- If $f'(x) > 0$ for all x in (a, b), f is increasing on (a, b).
- If $f'(x) < 0$ for all x in (a, b), f is decreasing on (a, b).

Mini-Quiz 7/19:

Let $f(x)=x^3$, and $0 \le x \le 1$.

(a) Find the slope of the secant line connecting the points $(0,0)$ and $(1,1)$.

(b) Find a number c in $(0,1)$ such that $f'(c)$ is equal to the slope of the secant line you computed in part (a). Explain why such a number must exist in $(0,1)$.

(c) Find the equation of the tangent line at $x=c$.

Exampl

SOLU

Accordi

$f(x)$ is

and $f(x)$ is