4. Use logarithmic differentiation to find the derivative of

$$y = \frac{\sqrt{x+3}\,(x^2+4)^2 \ln(3x)\,\sin x}{(x^2-1)^{\frac{5}{3}}(x^2+3x+4)^3(x+1)^2 e^x}$$

$\ln(AB) = \ln A + \ln B$

$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

$\ln(A^k) = k \ln A$

SOLUTION: $= \ln(\ \sim\ )$

$$\ln y = \ln\sqrt{x+3} + \ln(x^2+4)^2 + \ln(\ln(3x)) + \ln(\sin x) - \ln(x^2-1)^{\frac{5}{3}} - \ln(x^2+3x+4)^3 - \ln(x+1)^2 - \ln(e^x)$$

$$= \frac{1}{2}\ln(x+3) + 2\ln(x^2+4) + \ln(\ln(3x)) + \ln(\sin x) - \frac{5}{3}\ln(x^2-1) - 3\ln(x^2+3x+4) - 2\ln(x+1) - x$$

$\frac{1}{y}\frac{d}{d}$

$\frac{d}{dx}$

$\ln x)$

$n(e^{x})$

$in\, x)$

$-\, x$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\ln y = \frac{1}{2}\frac{1}{x+3} + 2\frac{1}{x^2+4}\cdot 2x + \frac{1}{x\ln(3x)} + \frac{1}{\sin x}\cos x$$

$$- \frac{5}{3}\cdot\frac{1}{x^2-1}\cdot 2x - 3\cdot\frac{1}{x^2+3x+4}\cdot(2x+3) - 2\cdot\frac{1}{x+1} - 1$$

$$\boxed{\frac{dy}{dx} = y\left[\frac{1}{2(x+3)} + \frac{4x}{x^2+4} + \frac{1}{x\ln(3x)} + \cot x - \frac{10x}{3(x^2-1)} - \frac{3(2x+3)}{x^2+3x+4} - \frac{2}{x+1} - 1\right]}$$

$$\frac{d}{dx}\ln(\ln(3x)) = \frac{1}{\ln(3x)}\cdot\frac{d}{dx}\ln(3x)$$

$$= \frac{1}{\ln(3x)}\cdot\frac{1}{3x}\cdot\frac{d}{dx}(3x)$$

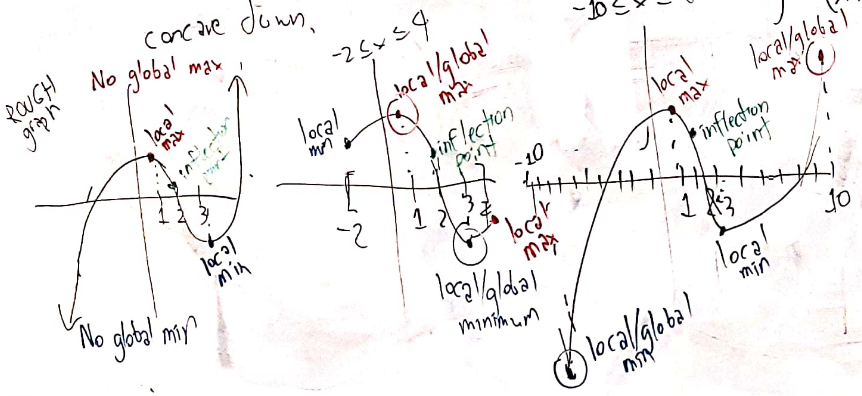
$$= \frac{1}{\ln(3x)}\cdot\frac{1}{\cancel{3}x}\cdot\cancel{3}$$

$$= \frac{1}{x\ln(3x)}$$

5. Determine where the function

$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$

is increasing and decreasing, and where it is concave up and concave down.



SOLUTION

$f'(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 - 2x^2 + 3x + 4\right)$

$= x^2 - 4x + 3$

$f''(x) = \frac{d}{dx} f'(x)$

$= \frac{d}{dx}\left(x^2 - 4x + 3\right)$

$= 2x - 4$

Increasing/decreasing

Set $f'(x) = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x - 1 = 0, \quad x - 3 = 0$

Critical points $x = 1, \; x = 3$

+		−		+
0	1	2	3	4

Test points

Therefore, $f(x)$ is increasing on $(-\infty, 1) \cup (3, \infty)$ and decreasing on $(1, 3)$.

$f'(0) =$

$=$

$=$

$f'(2) =$

$=$

$=$

$f'(4)$

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sing

$= 0$

$= 0$

$3 = 0$

3

$+$

4

$)\cup(3,\infty)$

$$f'(0) = 0^2 - 4\cdot 0 + 3$$
$$= 0 - 0 + 3$$
$$= 3$$
$$> 0$$

$$f'(2) = 2^2 - 4\cdot 2 + 3$$
$$= 4 - 8 + 3$$
$$= -1$$
$$< 0$$

$$f'(4) = 4^2 - 4\cdot 4 + 3$$
$$= 16 - 16 + 3$$
$$= 3$$
$$> 0$$

Concave up/Concave down

Set $f''(x) = 0$

$$2x - 4 = 0$$
$$2x = 4$$
$$x = 2$$

x-value of inflection point

$-$ $+$

1 2 3

Therefore, $\boxed{f(x) \text{ is concave down on } (-\infty, 2) \text{ and concave up on } (2,\infty)}$

$$f''(1) = 2\cdot 1 - 4$$
$$= -2$$
$$< 0$$

$$f''(3) = 2\cdot 3 - 4$$
$$= 2$$
$$> 0$$

When finding:

- Increasing/decreasing behavior
  Plug test points into $f'(x)$
- Concave up/Concave down behavior
  Plug test points into $f''(x)$
- Local/Global Extrema
  Plug critical points into $f(x)$ "Original Function"
- Inflection Points
  Plug x-coordinate of inflection point into $f(x)$

3 Consider $f(x) = \ln x$ and $[1, e]$.

(a) Find the slope of the secant line connecting the points $(1, 0)$ and $(e, 1)$.

(b) Find the number c in $(1, e)$ such that $f'(c)$ is equal to the slope of your secant line.

(c) Find the equation of the tangent line at $x = c$.

$(1,0)$ and $(e,1)$

(a)
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{e-1} = \boxed{\frac{1}{e-1}}$$

$x_1 = 1$
$x_2 = e$
$y_1 = 0$
$y_2 = 1$

(b) Set $f'(c) = m$

$$\frac{1}{c} = \frac{1}{e-1}$$

$$c = \boxed{e-1}$$

$f(a) = \ldots$
$f(b) = \ldots$

(b) Set $f'(c)=m$

$f(x)=\ln x$

$f'(x)=\frac{d}{dx}\ln x=\frac{1}{x}$

$f'(c)=\frac{1}{c}$

$$\frac{1}{c}=\frac{1}{e-1}$$

$$c=\boxed{e-1}$$

(c) $y-y_1=m(x-x_1)$

$$y-f(c)=f'(c)(x-c)$$

$$y-\ln(e-1)=\frac{1}{e-1}(x-(e-1))$$

$$y-\ln(e-1)=\frac{1}{e-1}x-1$$

$$\boxed{y=\frac{1}{e-1}x+\ln(e-1)-1}$$

$f(c)=\ln c$

$=\ln(e-1)$

$f'(c)=m$

$=\frac{1}{e-1}$