Mini-Quiz 6/30 Solution

Find the derivative of

$$y = \frac{\sqrt{3x^4+2}}{x^3-6}$$

quotient rule

$$\frac{d}{dx}\left(\sqrt{3x^4+2}\right) = \frac{d}{dx}\left((3x^4+2)^{\frac{1}{2}}\right)$$

chain rule

$$= \frac{1}{2}(3x^4+2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(3x^4+2)$$

$$= \frac{1}{2}(3x^4+2)^{-\frac{1}{2}} \cdot 12x^3$$

$$= \frac{6x^3}{\sqrt{3x^4+2}}$$

SOLUTION:

$$y' = \frac{d}{dx}\left(\frac{\sqrt{3x^4+2}}{x^3-6}\right)$$

$$= \frac{\frac{d}{dx}\left(\sqrt{3x^4+2}\right)(x^3-6) - \sqrt{3x^4+2}\,\frac{d}{dx}(x^3-6)}{(x^3-6)^2}$$

$$= \boxed{\frac{\frac{6x^3}{\sqrt{3x^4+2}}(x^3-6) - \sqrt{3x^4+2}\cdot(3x^2)}{(x^3-6)^2}}$$

Scanné avec CamScanner

4.6 Implicit Functions and Implicit Differentiation

Implicit differentiation helps us find $\frac{dy}{dx}$ even if we cannot explicitly solve for y in an algebraic equation, e.g.

$$5x^2 + 5y^2 - 6\sqrt{2}\,xy = 8.$$

Examp

SOLUTI

Example: Find $\frac{dy}{dx}$ if $x^2+y^2=1$.

SOLUTION:

$$x^2 + y^2 = 1$$

$y = y(x)$

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\longrightarrow 2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

Example: Find $\frac{dy}{dx}$ if $5x^2 + 5y^2 - 6\sqrt{2}\,xy = 8$.

SOLUTION:

$$\frac{d}{dx}\left(5x^2 + 5y^2 - 6\sqrt{2}\,xy\right) = \frac{d}{dx}(8)$$

$$10x + 10y\frac{dy}{dx} - \boxed{6\sqrt{2}\frac{d}{dx}(xy)} = 0$$

$$10x + 10y\frac{dy}{dx} - \boxed{6\sqrt{2}\left(y + x\frac{dy}{dx}\right)} = 0$$

$$10x + 10y\frac{dy}{dx} - 6\sqrt{2}y - 6\sqrt{2}x\frac{dy}{dx} = 0$$

$$\frac{d}{dx}(xy) = \frac{d}{dx}(x)\,y + x\frac{d}{dx}(y)$$

$$= 1y + x\frac{dy}{dx}$$

$$= y + x\frac{dy}{dx}$$

$10x - 6\sqrt{}$

$10x - 6\sqrt{2}$

$$10x - 6\sqrt{2}y + 10y\frac{dy}{dx} - 6\sqrt{2}x\frac{dy}{dx} = 0$$

$$10x - 6\sqrt{2}y + \frac{dy}{dx}\cdot(10y - 6\sqrt{2}x) = 0$$

$$\frac{dy}{dx}\cdot(10y - 6\sqrt{2}x) = -10x + 6\sqrt{2}y$$

$$\boxed{\frac{dy}{dx} = \frac{-10x + 6\sqrt{2}y}{10y - 6\sqrt{2}x}}$$

Example: Find $\frac{dy}{dx}$ when $y^2 = x^3$, assuming $x > 0$ and $y > 0$.

SOLUTION:

$$y^2 = x^3$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^3)$$

$$2y\frac{dy}{dx} = 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{2y}}$$

Optional step: Solve for y.

$$\sqrt{y^2} = \sqrt{x^3}$$

$$y = \sqrt{x^3}$$

$$y = x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

$$= \frac{3x^2}{2x^{\frac{3}{2}}}$$

$$= \frac{3}{2}x^{2-\frac{3}{2}}$$

Derivative of $y = x^{\frac{3}{2}}$

$$= \boxed{\frac{3}{2}x^{\frac{1}{2}}} = \frac{dy}{dx}$$

Related Rates

Example: If $x^2+y^3=1$, find $\frac{dy}{dt}$ at $x=\sqrt{\frac{7}{8}}$ if $\frac{dx}{dt}=2$.

Remark

$x=x(t)$ $\quad \frac{dx}{dt}$

$y=y(t)$ $\quad \frac{dy}{dt}$

SOLUTION: $x^2+y^3=1$

$$\frac{d}{dt}(x^2+y^3)=1$$

$$\frac{d}{dt}(x^2)+\frac{d}{dt}(y^3)=\frac{d}{dt}(1)$$

$$2x\frac{dx}{dt}+3y^2\frac{dy}{dt}=0$$

$$2x\frac{dx}{dt}+3y^2\frac{dy}{dt}=0.$$

$$3y^2\frac{dy}{dt}=-2x\frac{dx}{dt}$$

$$\frac{dy}{dt}=-\frac{2x}{3y^2}\frac{dx}{dt}=\boxed{-\frac{2\sqrt{\frac{7}{8}}}{3\left(\frac{1}{2}\right)^2}\cdot 2}$$

$$=\boxed{-\frac{16}{3}\sqrt{\frac{7}{8}}}=\boxed{\frac{-4\sqrt{14}}{3}}$$

$x^2+y^3=1$

$$\sqrt[3]{y^3}=\sqrt[3]{1-x^2}$$

$$y=\sqrt[3]{1-x^2}$$

$$=\sqrt[3]{1-\left(\sqrt{\frac{7}{8}}\right)^2}$$

$$=\sqrt[3]{1-\frac{7}{8}}$$

$$=\sqrt[3]{\frac{1}{8}}=\frac{\sqrt[3]{1}}{\sqrt[3]{8}}=\frac{1}{2}$$

$$\left(y=\frac{1}{2}\right)$$

4.7 Higher Derivatives

Let f be a function.

Then:
- f' is the first derivative of f.
- f'' is the second derivative of f.
- f''' is the third derivative of f.
- $f^{(4)}$ is the fourth derivative of f.
- $f^{(5)}$ is the fifth derivative of f

... and so on...

Example: Find the n^{th} derivatives of

$$f(x) = x^5$$

for $n = 1, 2, 3, 4, \ldots$

SOLUTION:

$$f'(x) = \frac{d}{dx}(x^5) = \boxed{5x^4}$$

$$f''(x) = \frac{d}{dx}(5x^4) = \boxed{20x^3}$$

$$f'''(x) = \frac{d}{dx}(20x^3) = \boxed{60x^2}$$

$$f^{(4)}(x) = \frac{d}{dx}(60x^2) = \boxed{120x}$$

$$f^{(5)}(x) = \frac{d}{dx}(120x) = \boxed{120}$$

$$f^{(6)}(x) = \frac{d}{dx}(120) = 0$$

$$f^{(7)}(x) = f^{(8)}(x) = \cdots = 0$$

Example: Find the second d

$$f(x) = \sqrt{x}$$

for $x \geq 0$.

SOLUTION:

$$f'(x) = \frac{d}{dx}(\sqrt{x}$$

$$= \frac{d}{dx}(x^{\frac{1}{2}}$$

$$= \frac{1}{2}x$$

$$= \frac{1}{2x^{\frac{1}{2}}}$$

$$= \frac{1}{2\sqrt{x}}$$

Example: Find the second derivative of

$$f(x)=\sqrt{x}$$

for $x \geq 0$.

SOLUTION: $f'(x)=\frac{d}{dx}(\sqrt{x})$

$$=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

$$=\frac{1}{2}x^{-\frac{1}{2}}$$

$$=\frac{1}{2x^{\frac{1}{2}}}$$

$$=\frac{1}{2\sqrt{x}}$$

$$f''(x)=\frac{d}{dx}(f'(x))=\frac{d}{dx}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{2}\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{2}\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$=\boxed{-\frac{1}{4}x^{-\frac{3}{2}}}$$

$$=\boxed{-\frac{1}{4x^{\frac{3}{2}}}}$$

Quotient Rule Solution

$$f''(x)=\frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)$$

$$=\frac{\frac{d}{dx}(1)(2\sqrt{x})-1\cdot\frac{d}{dx}(2\sqrt{x})}{(2\sqrt{x})^2}$$

$$=\frac{0\cdot 2\sqrt{x}-1\cdot\left[\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x}}\right]}{4x}$$

$$=\frac{-\frac{1}{\sqrt{x}}}{4x}=-\frac{1}{x^{\frac{1}{2}}}\cdot\frac{1}{4x}=\boxed{-\frac{1}{4x^{\frac{3}{2}}}}$$

Mini-Quiz 7/5

Use implicit differentiation to find $\frac{dy}{dx}$ when

$$x^2 y^2 = 1.$$