## Mini-Quiz 7/5 Solution

Use implicit differentiation to find $\frac{dy}{dx}$ when

$$x^2y^2 = 1.$$

SOLUTION:

$$x^2y^2 = 1$$

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^2)y^2 + x^2\frac{d}{dx}(y^2) = 0$$

$$2xy^2 + x^2\left(2y\frac{dy}{dx}\right) = 0$$

$$2x^2y\frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = \boxed{\frac{-2xy^2}{2x^2y}} =$$

## 4.8 Derivatives of Trigonometric Functions

**Theorem** The functions $\sin x$ and $\cos x$ are differentiable for all $x$, and

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x.$$

Scanné avec CamScanner

# 4.8 Derivatives of Trigonometric Functions

**Theorem** The functions $\sin x$ and $\cos x$ are differentiable for all $x$, and

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x.$$

1 Network Configuration
Connect To Wi-Fi Network
2 Choose This Apple TV
SPRL1340

**Theorem** Use the limit definition to prove

(a) $\frac{d}{dx}\sin x = \cos x$

(b) $\frac{d}{dx}\cos x = -\sin x$.

PROOF OF (a): Use the trig. identity

$$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

The difference quotient is

$$\frac{\sin(x+h) - \sin x}{h} = \frac{[\sin x \cos h + \cos x \sin h] - \sin x}{h}$$

$$= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \sin x \, \frac{\cos h - 1}{h} + \cos x \, \frac{\sin h}{h}$$

Therefo

$\frac{d}{dx}$ sin

Therefore,

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\left[\sin x\,\frac{\cos h - 1}{h} + \cos x\,\frac{\sin h}{h}\right]$$

$$= \sin x\,\boxed{\lim_{h\to 0}\frac{\cos h - 1}{h}} = 0 \quad + \cos x\,\boxed{\lim_{h\to 0}\frac{\sin h}{h}} = 1$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= \cos x .$$

Hint for the proof of (b):

The proof of (b) is similar to the proof of (a). Use the trig. identity

$$\cos(x+h) = \cos x \cos h - \sin x \sin h .$$

Example: Find the derivative of

$$f(x) = -4\sin x + \cos\frac{\pi}{6}$$

SOLUTION:

$$f'(x) = \frac{d}{dx}\left(-4\sin x + \cos\frac{\pi}{6}\right) = \frac{d}{dx}\left(-4\sin x\right) + \frac{d}{dx}\left(\cos\frac{\pi}{6}\right)$$

$$= -4\frac{d}{dx}\sin x + \frac{d}{dx}\left(\cos\frac{\pi}{6}\right)$$

This is a CONSTANT

$$= -4\cos x + 0$$

$$= \boxed{-4\cos x}$$

ample: Find the derivative of

$$y = \cos(x^2+1)$$

LUTION:

$$y' = \frac{d}{dx}\left(\cos(x^2+1)\right)$$

$$= -\sin(x^2+1) \cdot \frac{d}{dx}(x^2+1)$$

$$= \boxed{-\sin(x^2+1)\cdot 2x}$$

$$= \boxed{-2x\sin(x^2+1)}$$

Example: Find the derivative of

$$y = x^2\sin(3x) - \cos(5x)$$

SOLUTION:

$$y' = \frac{d}{dx}\left(x^2\sin(3x)\right) - \frac{d}{dx}\cos(5x)$$

$$= \left[\frac{d}{dx}(x^2)\sin(3x) + x^2\frac{d}{dx}\sin(3x)\right] - \left[-\sin(5x)\cdot 5\right]$$

$$= \boxed{\left[2x\sin(3x) + x^2(\cos(3x)\cdot 3)\right] + 5\sin(5x)}$$

$$= \boxed{2x\sin(3x) + 3x^2\cos(3x) + 5\sin(5x)}$$

The derivatives of the other trigonometric functions can be found using these identities.

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}.$$

Example: Prove $\frac{d}{dx}\tan x = \sec^2 x$

SOLUTION:

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$= \frac{\frac{d}{dx}(\sin x)\cdot \cos x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

In summary, the derivatives of the six trigonometric functions are

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\csc x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sec x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x$$

In summary, the derivatives of the six trigonometric functions are:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Example: Compute the derivatives of:

(a): $\tan(x^2) = \tan x^2$ (b): $\tan^2 x = (\tan x)^2$

SOLUTION:

(a): $\frac{d}{dx}\tan(x^2) = \sec^2(x^2)\cdot\frac{d}{dx}(x^2)$

$= \boxed{\sec^2(x^2)\cdot 2x}$

$= \boxed{2x\sec^2(x^2)}$

(b): $\frac{d}{dx}\tan^2 x = 2\tan x\cdot\frac{d}{dx}(\tan x)$

$= \boxed{2\tan x\cdot\sec^2 x}$

Example: Find the equation of the tangent line of

$$f(x) = \tan x$$

at $x = \frac{\pi}{4}$.

SOLUTION:

$$f'(x) = \frac{d}{dx}\tan x = \sec^2 x$$

At $x = \frac{\pi}{4}$,

$$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{2}{4}} = \frac{4}{2} = 2$$

$$f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Equation of the tangent line

$$y - y_1 = m(x - x_1)$$

$$y - f\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$

$$\boxed{y - 1 = 2\left(x - \frac{\pi}{4}\right)}$$

$$y - 1 = 2x - \frac{\pi}{2}$$

$$\boxed{y = 2x + \left(1 - \frac{\pi}{2}\right)}$$

**Example:** Find the derivative of

$$f(x) = \sec\left(\sqrt{x^2+1}\right)$$

SOLUTION:

$$f'(x) = \frac{d}{dx}\sec\left(\sqrt{x^2+1}\right)$$

$$= \sec\left(\sqrt{x^2+1}\right)\tan\left(\sqrt{x^2+1}\right)\cdot\frac{d}{dx}\left(\sqrt{x^2+1}\right)$$

$$= \boxed{\sec\left(\sqrt{x^2+1}\right)\tan\left(\sqrt{x^2+1}\right)\cdot\frac{x}{\sqrt{x^2+1}}}$$

$$\frac{d}{dx}\sqrt{x^2+1} = \frac{d}{dx}\left(\left(x^2+1\right)^{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left(x^2+1\right)$$

$$= \frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x$$

$$= \frac{x}{\sqrt{x^2+1}}$$

# Mini-Quiz 7/6

Find the derivative of

$$y = \sin(\cos(3x)).$$