Wrap up Section 4.5

Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and its derivative is given by

$$(f \circ g)'(x) = f'(g(x))\, g'(x).$$

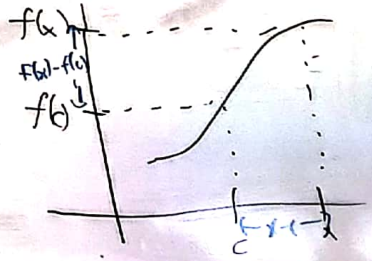
Proof It suffices to establish

$$(f \circ g)'(c) = f'(g(c))\, g'(c)$$

at any fixed $x = c$.

We use the limit definition of the derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$



Case 1: $g(x) \neq g(c)$

So $g(x)-g(c) \neq 0$. So we can multiply and divide by $g(x)-g(c)$.

We have

$$(f\circ g)'(c) = \lim_{x\to c} \frac{f(g(x))-f(g(c))}{x-c}$$

$$= \lim_{x\to c}\left(\frac{f(g(x))-f(g(c))}{x-c}\cdot\frac{g(x)-g(c)}{g(x)-g(c)}\right)$$

SWAP

$$= \lim_{x\to c}\left(\frac{f(g(x))-f(g(c))}{g(x)-g(c)}\cdot\frac{g(x)-g(c)}{x-c}\right)$$

$$= \lim_{x\to c}\frac{f(g(x))-f(g(c))}{g(x)-g(c)}\cdot\lim_{x\to c}\frac{g(x)-g(c)}{x-c}$$

As $x \to c$ and g is continuous, we also get $g(x)\to g(c)$

$$= \lim_{g(x)\to g(c)}\frac{f(g(x))-f(g(c))}{g(x)-g(c)}\cdot\lim_{x\to c}\frac{g(x)-g(c)}{x-c}$$

$$= f'(g(c))\,g'(c),$$

which completes the proof for this case.

...

Case 2: $g(x) = g(c)$

$$(f \circ g)'(c) = \lim_{x \to c} \frac{f(g(x)) - f(g(c))}{x - c}$$

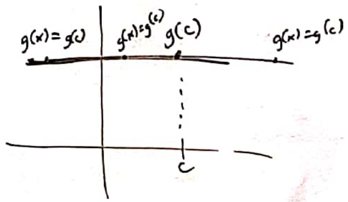
$$= \lim_{x \to c} \frac{f(g(c)) - f(g(c))}{x - c}$$

$$= \lim_{x \to c} \frac{0}{x - c}$$

$$= \lim_{x \to c} 0$$

$$= 0.$$

At the same time, since $g(x) = g(c)$ for all x, it follows that g is a constant function.



So $g'(x) = 0$.

In particular, at $x = c$, $g'(c) = 0$.

So $f'(g(c))g'(c) = f'(g(c)) \cdot 0$
$= 0.$

Therefore, $(f \circ g)'(c) = 0 = f'(g(c))g'(c)$, completing the proof for this case.

Outline of Midterm Exam on 7/11

Start: 9:05 AM
Stop: 10:25 AM

1 hour, 20 minutes

Gradescope submission deadline: 10:45 AM

Five questions, but some of them will have multiple parts (e.g. (a), (b), (c), ...)

What to bring

* Photo ID
  - set it on your desk during the exam

  No photo ID = zero score on midterm

* Pencil or pen and eraser
* Scientific or graphing calculator.
* 8×11 cheat sheet, two sided, handwritten

Scanné avec CamScanner

Content

One question will be on filling in limit tables

$f(x) =$ some function

x	$f(x)$
0.49	
0.499	
0.4999	
0.49999	

Use a calculator

$\lim_{x \to \frac{1}{2}^-} f(x) = \ldots\ldots$

Look at Section 3.1,
Exercises 1-32
page 111

Another question:

Evaluate limits

Multiple parts: (a), (b), (c), ..., (h)

Look at Sections 3.3, (p. 123)
Exercises 1-24
and Section 3.4, (p. 129)
Exercises 1-21

ignore graphing calculator parts of the exercises

Two questions:

Compute derivatives

Multiple parts each question: (a), (b), (c), ..., (f)

up to Section 4.5

- limit definition of derivative
- power rule
- product rule
- quotient rule
- chain rule

One question on proving a theorem:
could be *ONE* of the following

- Power rule for positive integers using the binomial theorem
- Chain rule
- Product rule
- Quotient rule with either:
  - limit definition
  - product and chain rule
- Trigonometric limits $\lim_{x \to 0} \frac{\sin x}{x} = 1$, $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$.

Mini-Quiz 7/7

In at least three complete sentences, describe your own strategy of preparing for this midterm.

Content

One question w

limit tables

x	f(x)
0.49	
0.499	
0.4999	
0.49999	

$\lim_{x \to \frac{1}{2}^-} f(x) =$

Look at Se

Exercises

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