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Name: SOLUTION Student ID: _____

UC Riverside
MATH 007A
Summer 2022
Midterm Examination

Write your name and student ID number on every page. This ensures that there will be no mix-up between you and your classmates when your work is being graded.

Show all work. You are *not* required to simplify your final answers, but you must perform all required computations involving limits and/or derivatives in your solution. Finally, wherever you have shown substantial work, please box your final answers!

You are allowed to use one two-sided 8×11 cheat sheet with anything you have handwritten, provided that you have already uploaded a scanned copy of it onto Gradescope. You may also use a calculator on questions that require it. All other resources are prohibited.

Please have your photo ID on your desk for the duration of the exam. Acceptable forms of photo identification include your UCR ID card, driver license, government-issued ID, or passport. The instructor will walk around the classroom during the exam to inspect your photo ID and your cheat sheet.

This should go without saying, but please uphold your academic honesty and integrity while taking this exam. Any instance of cheating could lead to an automatic zero score on this exam or, if the violation is severe enough, a failing grade in the entire course.

You have 1 hour and 20 minutes to complete this midterm exam. No time extensions will be granted except for reasons approved by the Student Disability Resource Center. *Do not start the exam until your instructor tells everyone to do so.*

If you need to use the restroom during the exam, you must ask the instructor for permission and leave your phone on your desk. You cannot make up any time you missed while outside the classroom. Do not stay outside the classroom for more than five (5) minutes at a time, and do not use the restroom more than twice during the exam; otherwise, the instructor will take your exam and dismiss you early, and you will only be graded for the work done.

In order for your exam score to count, you must sign on the dotted line below, which indicates that you have read and agreed to these policies. Submit this page on Gradescope with your exam once you are finished.

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Name:_____ Student ID:_____

(27pts) 1. You may use a scientific or graphing calculator for this question. If you do not have a calculator with you, you may use a calculator app from your phone or tablet. (Make sure your phone or tablet is set on airplane mode during the exam!) Although you are normally asked to show your work, you do *not* need to write down any arithmetic for this question only.

Consider the function $f(x) = \dfrac{x+4}{(x-1)(x-2)^2}$.

Fill in the tables, and round your approximations to four decimal places. Investigate all required limits. If a limit does not exist, write "DNE".

(4pts) (a) Fill in the following table and conclude $\lim_{x\to 0^-} f(x)$.

x	$f(x)$
-0.1	-0.8040
-0.01	-0.9778
-0.001	-0.9978

$\lim_{x\to 0^-} f(x) =$ -1

(4pts) (b) Fill in the following table and conclude $\lim_{x\to 0^+} f(x)$.

x	$f(x)$
0.1	-1.2619
0.01	-1.0028
0.001	-1.0023

$\lim_{x\to 0^+} f(x) =$ -1

(1pt) (c) Use the limits you found in parts (a) and (b) to conclude $\lim_{x\to 0} f(x)$.

$\lim_{x\to 0} f(x) =$ -1

Question 1 continues on the next page.

PLEASE WRITE YOUR NAME AND STUDENT ID NUMBER ON **EVERY** PAGE.

Name:_____ Student ID:_____

Question 1 continued.

(4pts) (d) Fill in the following table and conclude $\lim_{x\to 1^-} f(x)$.

x	$f(x)$
0.9	-40.4959
0.99	-489.1677
0.999	-4989.0170

$\lim_{x\to 1^-} f(x) =$ $-\infty$

(4pts) (e) Fill in the following table and conclude $\lim_{x\to 1^+} f(x)$.

x	$f(x)$
1.1	62.9630
1.01	511.1723
1.001	5011.0170

$\lim_{x\to 1^+} f(x) =$ $+\infty$

(1pt) (f) Use the limits you found in parts (d) and (e) to conclude $\lim_{x\to 1} f(x)$.

$\lim_{x\to 1} f(x) =$ D.N.E.

(4pts) (g) Fill in the following table and conclude $\lim_{x\to 2^-} f(x)$.

x	$f(x)$
1.9	655.5556
1.99	60505.0505
1.999	6005005.0050

$\lim_{x\to 2^-} f(x) =$ $+\infty$

(4pts) (h) Fill in the following table and conclude $\lim_{x\to 2^+} f(x)$.

x	$f(x)$
2.1	554.5455
2.01	59504.9505
2.001	5995004.9951

$\lim_{x\to 2^+} f(x) =$ $+\infty$

(1pt) (i) Use the limits you found in parts (g) and (h) to conclude $\lim_{x\to 2} f(x)$.

$\lim_{x\to 2} f(x) =$ $+\infty$

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Name:_____ Student ID:_____

(24pts) 2. Compute the following limits. Express each final answer as either an integer or fraction. (All correct answers for this question are either integers or fractions.)

(3pts) (a) $\lim_{x\to 2} \dfrac{x^5+3}{x+5}$

$$\lim_{x\to 2} \frac{x^5+3}{x+5} = \frac{2^5+3}{2+5} = \frac{32+3}{2+5} = \frac{35}{7} = \boxed{5}$$

(3pts) (b) $\lim_{x\to\infty} \dfrac{x^2-1}{x^2}$

$$\lim_{x\to\infty} \frac{x^2-1}{x^2} = \lim_{x\to\infty} \frac{\frac{x^2-1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1} = \frac{1-0}{1} = \boxed{1}$$

(3pts) (c) $\lim_{x\to 3} \dfrac{2x^2-18}{x^2-x-6}$

$$\lim_{x\to 3} \frac{2x^2-18}{x^2-x-6} = \lim_{x\to 3} \frac{2(x^2-9)}{x^2-x-6} = \lim_{x\to 3} \frac{2(x+3)(x-3)}{(x-3)(x+2)} = \lim_{x\to 3} \frac{2(x+3)}{x+2} = \frac{2(3+3)}{3+2} = \frac{2\cdot 6}{5} = \boxed{\frac{12}{5}}$$

(3pts) (d) $\lim_{x\to 0} \dfrac{2-\sqrt{4-x^2}}{x^2}$

$$\lim_{x\to 0} \frac{2-\sqrt{4-x^2}}{x^2} = \lim_{x\to 0} \frac{2-\sqrt{4-x^2}}{x^2} \cdot \frac{2+\sqrt{4-x^2}}{2+\sqrt{4-x^2}} = \lim_{x\to 0} \frac{4-(4-x^2)}{x^2(2+\sqrt{4-x^2})} = \lim_{x\to 0} \frac{x^2}{x^2(2+\sqrt{4-x^2})} = \lim_{x\to 0} \frac{1}{2+\sqrt{4-x^2}} = \frac{1}{2+\sqrt{4-0^2}} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

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Name:_____ Student ID:_____

Question 2 continued.

(3pts) (e) $\lim_{x\to\infty} \dfrac{1-e^x}{2-e^x}$

$$\lim_{x\to\infty} \frac{1-e^x}{2-e^x} = \lim_{x\to\infty} \frac{1-e^x}{2-e^x}\frac{e^{-x}}{e^{-x}}$$
$$= \lim_{x\to\infty} \frac{e^{-x}-1}{2e^{-x}-1}$$
$$= \frac{0-1}{2\cdot 0-1}$$
$$= \boxed{1}$$

(3pts) (f) $\lim_{x\to 4} \sin\left(\dfrac{\pi x}{24}\right)$

$$\lim_{x\to 4} \sin\left(\frac{\pi x}{24}\right) = \sin\left(\frac{\pi\cdot 4}{24}\right)$$
$$= \sin\left(\frac{\pi}{6}\right)$$
$$= \boxed{\frac{1}{2}}$$

(3pts) (g) $\lim_{x\to 0} \dfrac{\sin(3x)(1-\cos(2x))}{6x^2}$

$$\lim_{x\to 0} \frac{\sin(3x)(1-\cos(2x))}{6x^2} = \lim_{x\to 0}\left(\frac{\sin(3x)}{3x}\,\frac{1-\cos(2x)}{2x}\right)$$
$$= \lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\lim_{x\to 0}\frac{1-\cos(2x)}{2x}$$
$$= 1\cdot 0$$
$$= \boxed{0}$$

(3pts) (h) $\lim_{x\to 0} x^4\cos\left(\dfrac{1}{x}\right)$

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$
$$-x^4 \le x^4\cos\left(\frac{1}{x}\right) \le x^4$$

$\lim_{x\to 0}(-x^4) = 0$

$\lim_{x\to 0} x^4 = 0$

By the Sandwich Theorem,

$$\lim_{x\to 0} x^4\cos\left(\frac{1}{x}\right) = \boxed{0}$$

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Name:_____ Student ID:_____

(21pts) 3. Prove the following limits. (We proved these in lecture for Section 3.4.)

~~(12pts)~~ (14pts) (b) ~~(a)~~ $\lim_{x \to 0} \frac{\sin x}{x} = 1$

See the lecture notes of our meeting on 6/23/2022.

~~(9pts)~~ (7pts) (a) ~~(b)~~ $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

See the lecture notes of our meeting on 6/23/2022.

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Name:_____ Student ID:_____

(24pts) 4. Compute the derivatives of the following functions.

(3pts) (a) $f(x) = 4x^5 + 3x^4 + 2x^3 + x^2$

$$f'(x) = \frac{d}{dx}\left(4x^5 + 3x^4 + 2x^3 + x^2\right)$$
$$= \boxed{20x^4 + 12x^3 + 6x^2 + 2x}$$

(3pts) (b) $f(x) = \frac{1}{x^5} + \frac{2}{x^4} + \frac{3}{x^3} + \frac{4}{x^2}$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{x^5} + \frac{2}{x^4} + \frac{3}{x^3} + \frac{4}{x^2}\right)$$
$$= \frac{d}{dx}\left(x^{-5} + 2x^{-4} + 3x^{-3} + 4x^{-2}\right)$$
$$= -5x^{-6} - 8x^{-5} - 9x^{-4} - 8x^{-3}$$
$$= \boxed{-\frac{5}{x^6} - \frac{8}{x^5} - \frac{9}{x^4} - \frac{8}{x^3}}$$

(3pts) (c) $f(x) = 2\sqrt{x} + 3\sqrt[3]{x} + 5x^{\frac{8}{5}}$

$$f'(x) = \frac{d}{dx}\left(2\sqrt{x} + 3\sqrt[3]{x} + 5x^{\frac{8}{5}}\right)$$
$$= \frac{d}{dx}\left(2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 5x^{\frac{8}{5}}\right)$$
$$= x^{-\frac{1}{2}} + x^{-\frac{2}{3}} + 8x^{\frac{3}{5}}$$
$$= \boxed{\frac{1}{\sqrt{x}} + \frac{1}{x^{\frac{2}{3}}} + 8x^{\frac{3}{5}}}$$

(3pts) (d) $f(x) = 2x^{\pi} + \pi x^2$

$$f'(x) = \frac{d}{dx}\left(2x^{\pi} + \pi x^2\right)$$
$$= \boxed{2\pi x^{\pi-1} + 2\pi x}$$

PLEASE WRITE YOUR NAME AND STUDENT ID NUMBER ON **EVERY** PAGE.

Name:_____ Student ID:_____

Question 4 continued.

(3pts) (e) $f(x) = \frac{x}{\pi} + \frac{\pi}{2} + \frac{\pi}{x} + \frac{2}{\pi}$

$$f'(x) = \frac{d}{dx}\left(\frac{x}{\pi} + \frac{\pi}{2} + \frac{\pi}{x} + \frac{2}{\pi}\right)$$
$$= \frac{1}{\pi} + 0 - \frac{\pi}{x^2} + 0$$
$$= \boxed{\frac{1}{\pi} - \frac{\pi}{x^2}}$$

(3pts) (f) $f(x) = \frac{x^3}{15} - \frac{x^4}{20} + \frac{2}{15}$

$$f'(x) = \frac{d}{dx}\left(\frac{x^3}{15} - \frac{x^4}{20} + \frac{2}{15}\right)$$
$$= \frac{3x^2}{15} - \frac{4x^3}{20} + 0$$
$$= \boxed{\frac{x^2}{5} - \frac{x^3}{5}}$$

(3pts) (g) $f(x) = 20x^3 - 4x^6 + 9x^8$

$$f'(x) = \frac{d}{dx}(20x^3 - 4x^6 + 9x^8)$$
$$= \boxed{60x^2 - 24x^5 + 72x^7}$$

(3pts) (h) $f(x) = ax^3 + bx$, where a, b are constants

$$f'(x) = \frac{d}{dx}(ax^3 + bx)$$
$$= \boxed{3ax^2 + b}$$

PLEASE WRITE YOUR NAME AND STUDENT ID NUMBER ON **EVERY** PAGE.

Name:_____ Student ID:_____

(24pts) 5. Compute the derivatives of the following functions.

(4pts) (a) $f(x) = (x^2 + 5)(x^2 - 3)$

$$f'(x) = \frac{d}{dx}\left[(x^2+5)(x^2-3)\right]$$
$$= \frac{d}{dx}(x^2+5)(x^2-3) + (x^2+5)\frac{d}{dx}(x^2-3)$$
$$= \boxed{2x(x^2-3) + (x^2+5)\cdot 2x}$$

(4pts) (b) $f(x) = (3x^4 - 2x^2 + 1)(x^2 + 4x - 1)$

$$f'(x) = \frac{d}{dx}\left[(3x^4-2x^2+1)(x^2+4x-1)\right]$$
$$= \frac{d}{dx}(3x^4-2x^2+1)(x^2+4x-1) + (3x^4-2x^2+1)\frac{d}{dx}(x^2+4x-1)$$
$$= \boxed{(12x^3-4x)(x^2+4x-1) + (3x^4-2x^2+1)(2x+4)}$$

(4pts) (c) $f(x) = \dfrac{x^2+4}{x-2}$

$$f'(x) = \frac{d}{dx}\left[\frac{x^2+4}{x-2}\right]$$
$$= \frac{\frac{d}{dx}(x^2+4)(x-2) - (x^2+4)\frac{d}{dx}(x-2)}{(x-2)^2}$$
$$= \boxed{\frac{2x(x-2) - (x^2+4)\cdot 1}{(x-2)^2}}$$

PLEASE WRITE YOUR NAME AND STUDENT ID NUMBER ON **EVERY** PAGE.

Name:_____ Student ID:_____

Question 5 continued.

(4pts) (d) $f(x) = \dfrac{3x^2 - 2x + 1}{3x^2 + 2x - 1}$

$$f'(x) = \frac{d}{dx}\left[\frac{3x^2-2x+1}{3x^2+2x-1}\right]$$

$$= \frac{\frac{d}{dx}(3x^2-2x+1)(3x^2+2x-1) - (3x^2-2x+1)\frac{d}{dx}(3x^2+2x-1)}{(3x^2+2x-1)^2}$$

$$= \boxed{\frac{(6x-2)(3x^2+2x-1) - (3x^2-2x+1)(6x+2)}{(3x^2+2x-1)^2}}$$

(4pts) (e) $f(x) = \sqrt{2x^2 - x + 3}$

$$f'(x) = \frac{d}{dx}\left(\sqrt{2x^2-x+3}\right)$$

$$= \frac{d}{dx}\left((2x^2-x+3)^{\frac{1}{2}}\right)$$

$$= \frac{1}{2}(2x^2-x+3)^{-\frac{1}{2}}\frac{d}{dx}(2x^2-x+3)$$

$$= \boxed{\frac{1}{2\sqrt{2x^2-x+3}}(4x-1)}$$

(4pts) (f) $f(x) = (7x^3 + 4x)^{999}$

$$f'(x) = \frac{d}{dx}\left((7x^3+4x)^{999}\right)$$

$$= 999(7x^3+4x)^{998}\frac{d}{dx}(7x^3+4x)$$

$$= \boxed{999(7x^3+4x)^{998}(21x^2+4)}$$