

Quiz 1 solutions

1. Evaluate the indefinite integral

$$\int x \sqrt{x} \, dx.$$

Solution. Note that we can rewrite the integrand as

$$\begin{aligned} x \sqrt{x} &= x^1 x^{\frac{1}{2}} \\ &= x^{1+\frac{1}{2}} \\ &= x^{\frac{3}{2}}. \end{aligned}$$

So our antiderivative is

$$\begin{aligned} \int x \sqrt{x} \, dx &= \int x^{\frac{3}{2}} \, dx \\ &= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C \\ &= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C \\ &= \boxed{\frac{2}{5} x^{\frac{5}{2}} + C}, \end{aligned}$$

where C is an arbitrary constant. □

2. Evaluate the indefinite integral

$$\int (2x - 3) \left(\frac{1}{x} + 1 \right) \, dx.$$

Solution. Note that we can rewrite the integrand as

$$\begin{aligned} (2x - 3) \left(\frac{1}{x} + 1 \right) &= 2x \cdot \frac{1}{x} + 2x \cdot 1 - 3 \cdot \frac{1}{x} - 3 \cdot 1 \\ &= 2 + 2x - \frac{3}{x} - 3 \\ &= 2x - \frac{3}{x} - 1. \end{aligned}$$

So our antiderivative is

$$\begin{aligned} \int (2x - 3) \left(\frac{1}{x} + 1 \right) \, dx &= \int 2x - \frac{3}{x} - 1 \, dx \\ &= \int 2x \, dx - \int \frac{3}{x} \, dx - \int 1 \, dx \\ &= 2 \int x \, dx - 3 \int \frac{1}{x} \, dx - \int 1 \, dx \\ &= 2 \int x^1 \, dx - 3 \int x^{-1} \, dx - \int x^0 \, dx \\ &= 2 \left(\frac{x^{1+1}}{1+1} + C_1 \right) - 3(\ln |x| + C_2) - \left(\frac{x^{0+1}}{0+1} + C_3 \right) \\ &= 2 \left(\frac{x^2}{2} + C_1 \right) - 3(\ln |x| + C_2) - (x + C_3) \\ &= x^2 + 2C_1 - 3 \ln |x| - 3C_2 - x - C_3 \\ &= x^2 - x - 3 \ln |x| + (2C_1 - 3C_2 - C_3) \\ &= \boxed{x^2 - x - 3 \ln |x| + C}, \end{aligned}$$

where C_1, C_2, C_3 are all arbitrary constants and we let $C = 2C_1 - 3C_2 - C_3$. □