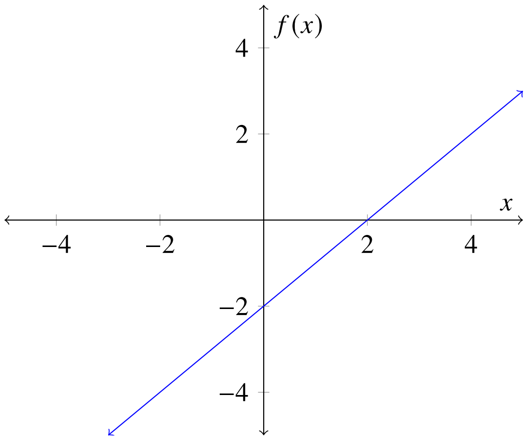
Ryan Ta

Quiz 2 solutions

1. Graph $f(x) = x - 2$. Using this graph, compute

$$\int_0^3 f(x)\,dx.$$

Solution. The graph of $f(x) = x - 2$ is given below.



Using the graph, we employ the formula for the area of a triangle to obtain

$$\begin{aligned}\int_0^2 f(x)\,dx &= \frac{1}{2} \cdot 2 \cdot (-2) \\ &= -2\end{aligned}$$

and

$$\begin{aligned}\int_2^3 f(x)\,dx &= \frac{1}{2} \cdot 1 \cdot 1 \\ &= \frac{1}{2}.\end{aligned}$$

So we have

$$\begin{aligned}\int_0^3 f(x)\,dx &= \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx \\ &= -2 + \frac{1}{2} \\ &= \boxed{-\frac{3}{2}},\end{aligned}$$

as desired. □

Question 2 solution is on the next page.

2. Let $f(x)$ be such that

$$\int_0^2 f(x)\,dx = -10,$$
$$\int_0^5 f(x)\,dx = 2.$$

Using Theorem 5.2.1 (see the original quiz), evaluate the following:

(a) $\int_2^5 f(x)\,dx$

Solution. By Rule 2 of Theorem 5.2.1, we can write

$$\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx.$$

Based on the given equation, our above equation becomes

$$2 = -10 + \int_2^5 f(x)\,dx.$$

Add 10 on both sides to conclude

$$\int_2^5 f(x)\,dx = \boxed{12},$$

as desired. □

(b) $\int_5^2 3f(x)\,dx$

Solution. By Rule 3 of Theorem 5.2.1, we can write

$$\int_5^2 3f(x)\,dx = -\int_2^5 3f(x)\,dx.$$

By Rule 5 of Theorem 5.2.1, we can write

$$\int_2^5 3f(x)\,dx = 3\int_2^5 f(x)\,dx.$$

Therefore, we have

$$\begin{aligned}\int_5^2 3f(x)\,dx &= -\int_2^5 3f(x)\,dx \\ &= -3\int_2^5 f(x)\,dx \\ &= -3 \cdot 12 \\ &= \boxed{-36},\end{aligned}$$

as desired. □