MATH 009B discussion
University of California, Riverside

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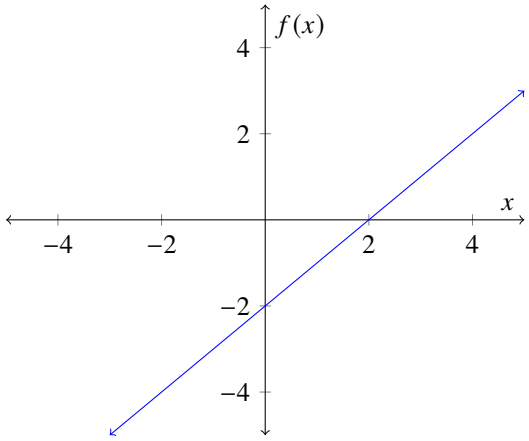
Quiz 3 solutions

1. Graph the function $f(x) = x - 2$. Using the graph, approximate $\int_0^3 f(x)\,dx$ using 3 rectangles placed using the right hand rule.

   Solution. The graph of $f(x) = x - 2$ is given below.



   For 3 rectangles, we have $\Delta x = \frac{3-0}{3} = 1$ and $x_{i+1} = 0 + i\Delta x = i$ for $i = 1, 2, 3$, and so we have

$$\begin{aligned} S_R(3) &= \sum_{i=1}^{3} f(x_{i+1})\Delta x \\ &= \sum_{i=1}^{3} f(i) \cdot 1 \\ &= \sum_{i=1}^{3} (i-2) \\ &= (1-2) + (2-2) + (3-2) \\ &= -1 + 0 + 1 \\ &= \boxed{0}, \end{aligned}$$

   as desired. □

2. Write a formula for the Right Riemann Sum $S_R(n)$ of $\int_0^3 f(x)\,dx$ with n rectangles.

   Solution. For n rectangles, we have $\Delta x = \frac{3-0}{n} = \frac{3}{n}$ and $x_{i+1} = 0 + i\Delta x = \frac{3i}{n}$ for $i = 1, \ldots, n$, and so we have

$$\begin{aligned} S_R(n) &= \sum_{i=1}^{n} f(x_{i+1})\Delta x \\ &= \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n} \\ &= \boxed{\sum_{i=1}^{n} \left(\frac{3i}{n} - 2\right)\frac{3}{n}}, \end{aligned}$$

   as desired. □