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Quiz 4 solutions

1. Compute

$$\frac{d}{dx}\int_{2x}^{3x} \tan(t)\, dt.$$

Solution. Using Theorem 5.2.1 (see the original quiz), we have

$$\begin{aligned}\int_{2x}^{3x} \tan(t)\, dt &= \int_{2x}^{0} \tan(t)\, dt + \int_{0}^{3x} \tan(t)\, dt \\ &= -\int_{0}^{2x} \tan(t)\, dt + \int_{0}^{3x} \tan(t)\, dt,\end{aligned}$$

By the Fundamental Theorem of Calculus, part 1, we compute

$$\begin{aligned}\frac{d}{dx}\int_{2x}^{3x} \tan(t)\, dt &= \frac{d}{dx}\left(-\int_{0}^{2x} \tan(t)\, dt + \int_{0}^{3x} \tan(t)\, dt\right) \\ &= -\frac{d}{dx}\int_{0}^{2x} \tan(t)\, dt + \frac{d}{dx}\int_{0}^{3x} \tan(t)\, dt \\ &= -\tan(2x)\frac{d}{dx}(2x) + \tan(3x)\frac{d}{dx}(3x) \\ &= \boxed{-2\tan(2x) + 3\tan(3x)},\end{aligned}$$

as desired. □

2. Evaluate

$$\int_{\pi}^{2\pi} \left(2\cos(x) - \frac{3}{x}\right) dx.$$

Solution. By the Fundamental Theorem of Calculus, part 2, we compute the indefinite integral

$$\begin{aligned}\int \left(2\cos(x) - \frac{3}{x}\right) dx &= 2\int \cos(x)\, dx - 3\int \frac{1}{x}\, dx \\ &= 2(\sin(x) + C_1) - (3\ln(|x|) + C_2) \\ &= 2\sin(x) - 3\ln(|x|) + (2C_1 - 3C_2) \\ &= 2\sin(x) - 3\ln(|x|) + C.\end{aligned}$$

Using this antiderivative, we also compute the definite integral

$$\begin{aligned}\int_{\pi}^{2\pi} \left(2\cos(x) - \frac{3}{x}\right) dx &= (2\sin(x) - 3\ln(|x|))\Big|_{\pi}^{2\pi} \\ &= (2\sin(2\pi) - 3\ln(|2\pi|)) - (2\sin(\pi) - 3\ln(|\pi|)) \\ &= (2\cdot 0 - 3\ln(2\pi)) - (2\cdot 0 - 3\ln(\pi)) \\ &= 3(\ln(\pi) - \ln(2\pi)) \\ &= 3\ln\left(\frac{\pi}{2\pi}\right) \\ &= \boxed{\ln\left(\frac{1}{8}\right)},\end{aligned}$$

as desired. □