Quiz 5: Trig Integrals

Instructions: write your solutions to the following two questions on separate sheets of paper. Show all work to receive credit. You will have 25 minutes to complete the Quiz and 10 minutes to upload your solutions to the Crowdmark assessment "Quiz 5" located in the Assignments tab of the **Discussion** iLearn.

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(1) Using Key Idea 6.3.1, evaluate the following indefinite integral

$$\int \cos^5 x \sin^3 x dx.$$

Key Idea 6.3.1 Integrals Involving Powers of Sine and Cosine

Consider $\int \sin^m x \cos^n x\, dx$, where m, n are nonnegative integers.

1. If m is odd, then $m = 2k+1$ for some integer k. Rewrite

$$\sin^m x = \sin^{2k+1} x = \sin^{2k} x \sin x = (\sin^2 x)^k \sin x = (1-\cos^2 x)^k \sin x.$$

Then

$$\int \sin^m x \cos^n x\, dx = \int (1-\cos^2 x)^k \sin x \cos^n x\, dx = -\int (1-u^2)^k u^n\, du,$$

where $u = \cos x$ and $du = -\sin x\, dx$.

2. If n is odd, then using substitutions similar to that outlined above we have

$$\int \sin^m x \cos^n x\, dx = \int u^m (1-u^2)^k\, du,$$

where $u = \sin x$ and $du = \cos x\, dx$.

3. If both m and n are even, use the power–reducing identities

$$\cos^2 x = \frac{1+\cos(2x)}{2} \quad \text{and} \quad \sin^2 x = \frac{1-\cos(2x)}{2}$$

to reduce the degree of the integrand. Expand the result and apply the principles of this Key Idea again.

(2) Using Key Idea 6.3.2, evaluate the following indefinite integral

$$\int \sec^6 x \tan x dx.$$

Key Idea 6.3.2 Integrals Involving Powers of Tangent and Secant

Consider $\int \tan^m x \sec^n x\, dx$, where m, n are nonnegative integers.

1. If n is even, then $n = 2k$ for some integer k. Rewrite $\sec^n x$ as

$$\sec^n x = \sec^{2k} x = \sec^{2k-2} x \sec^2 x = (1+\tan^2 x)^{k-1} \sec^2 x.$$

Then

$$\int \tan^m x \sec^n x\, dx = \int \tan^m x (1+\tan^2 x)^{k-1} \sec^2 x\, dx = \int u^m (1+u^2)^{k-1}\, du,$$

where $u = \tan x$ and $du = \sec^2 x\, dx$.

2. If m is odd, then $m = 2k+1$ for some integer k. Rewrite $\tan^m x \sec^n x$ as

$$\tan^m x \sec^n x = \tan^{2k+1} x \sec^n x = \tan^{2k} x \sec^{n-1} x \sec x \tan x = (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x.$$

Then

$$\int \tan^m x \sec^n x\, dx = \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x\, dx = \int (u^2-1)^k u^{n-1}\, du,$$

where $u = \sec x$ and $du = \sec x \tan x\, dx$.

3. If n is odd and m is even, then $m = 2k$ for some integer k. Convert $\tan^m x$ to $(\sec^2 x - 1)^k$. Expand the new integrand and use Integration By Parts, with $dv = \sec^2 x\, dx$.

4. If m is even and $n = 0$, rewrite $\tan^m x$ as

$$\tan^m x = \tan^{m-2} x \tan^2 x = \tan^{m-2} x (\sec^2 x - 1) = \tan^{m-2} \sec^2 x - \tan^{m-2} x.$$

So

$$\int \tan^m x\, dx = \underbrace{\int \tan^{m-2} \sec^2 x\, dx}_{\text{apply rule \#1}} - \underbrace{\int \tan^{m-2} x\, dx}_{\text{apply rule \#4 again}}.$$