Quiz 6: Trig Substitution

Instructions: write your solutions to the following two questions on separate sheets of paper. Show all work to receive credit. You will have 25 minutes to complete the Quiz and 10 minutes to upload your solutions to the Crowdmark assessment "Quiz 6" located in the Assignments tab of the **Discussion** iLearn.

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(1) Using an appropriate trigonemetric substitution, convert the following algebraic integrals into trigonemetric integrals without a square root. **DO NOT EVALUATE THE INTEGRAL**.

(a) $\displaystyle\int \frac{x^2 dx}{\sqrt{9+x^2}}$. (b) $\displaystyle\int x^2\sqrt{4-x^2}dx$. (c) $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}dx$.

(2) Write the following trigonemetric expressions in terms of x given the specified trig substitution.

(a) if $x = 2\sin\theta$, write $\frac{1}{2}\theta - \frac{1}{2}\cos\theta$ in terms of x.

(b) if $x = 3\tan\theta$, write $\ln|\sec\theta + \tan\theta|$ in terms of x.

(c) if $x = 1\sec\theta$, write $\cos\theta\sin\theta$ in terms of x.

Key Idea 6.4.1 Trigonometric Substitution

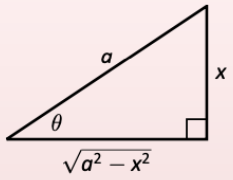
(a) For integrands containing $\sqrt{a^2-x^2}$:

Let $x = a\sin\theta$, $dx = a\cos\theta\, d\theta$

Thus $\theta = \sin^{-1}(x/a)$, for $-\pi/2 \le \theta \le \pi/2$.

On this interval, $\cos\theta \ge 0$, so

$\sqrt{a^2-x^2} = a\cos\theta$



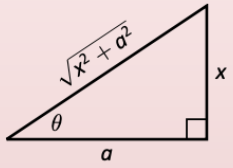
(b) For integrands containing $\sqrt{x^2+a^2}$:

Let $x = a\tan\theta$, $dx = a\sec^2\theta\, d\theta$

Thus $\theta = \tan^{-1}(x/a)$, for $-\pi/2 < \theta < \pi/2$.

On this interval, $\sec\theta > 0$, so

$\sqrt{x^2+a^2} = a\sec\theta$



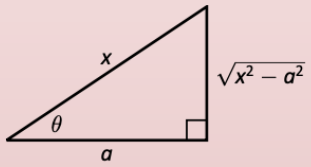
(c) For integrands containing $\sqrt{x^2-a^2}$:

Let $x = a\sec\theta$, $dx = a\sec\theta\tan\theta\, d\theta$

Thus $\theta = \sec^{-1}(x/a)$. If $x/a \ge 1$, then $0 \le \theta < \pi/2$; if $x/a \le -1$, then $\pi/2 < \theta \le \pi$.

We restrict our work to where $x \ge a$, so $x/a \ge 1$, and $0 \le \theta < \pi/2$. On this interval, $\tan\theta \ge 0$, so

$\sqrt{x^2-a^2} = a\tan\theta$



$\sin A$	=	$\frac{\text{opposite side}}{\text{hypotenuse}}$
$\cos A$	=	$\frac{\text{adjacent side}}{\text{hypotenuse}}$
$\tan A$	=	$\frac{\text{opposite side}}{\text{adjacent side}}$
$\csc A$	=	$\frac{\text{hypotenuse}}{\text{opposite side}}$
$\sec A$	=	$\frac{\text{hypotenuse}}{\text{adjacent side}}$
$\cot A$	=	$\frac{\text{adjacent side}}{\text{opposite side}}$