

QUIZ 7: PARTIAL FRACTION DECOMPOSITION

Instructions: write your solutions to the following two questions on separate sheets of paper. Show all work to receive credit. You will have 25 minutes to complete the Quiz and 10 minutes to upload your solutions to the Crowdmark assessment “Quiz 7” located in the Assignments tab of the **Discussion** iLearn.

- (1) **Set up** the decomposition for the following rational functions as a sum of elementary rational functions as described in Key Idea 6.5.1. Your answer should contain several unknowns.

$$(a) \frac{3x - 7}{x^2 - 4} \quad (b) \frac{x^2 - 2x + 3}{x^3 + 5x} \quad (c) \frac{x^3 - x + 1}{(x^2 + 2)(x^2 + 2)}.$$

- (2) Solve for all of the unknowns in the following equation.

$$\frac{3x^2 + 7x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}.$$

Key Idea 6.5.1 Partial Fraction Decomposition

Let $\frac{p(x)}{q(x)}$ be a rational function, where the degree of p is less than the degree of q .

- Linear Terms:** Let $(x - a)$ divide $q(x)$, where $(x - a)^n$ is the highest power of $(x - a)$ that divides $q(x)$. Then the decomposition of $\frac{p(x)}{q(x)}$ will contain the sum

$$\frac{A_1}{(x - a)} + \frac{A_2}{(x - a)^2} + \cdots + \frac{A_n}{(x - a)^n}.$$

- Quadratic Terms:** Let $x^2 + bx + c$ divide $q(x)$, where $(x^2 + bx + c)^n$ is the highest power of $x^2 + bx + c$ that divides $q(x)$. Then the decomposition of $\frac{p(x)}{q(x)}$ will contain the sum

$$\frac{B_1x + C_1}{x^2 + bx + c} + \frac{B_2x + C_2}{(x^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(x^2 + bx + c)^n}.$$

To find the coefficients A_i , B_i and C_i :

- Multiply all fractions by $q(x)$, clearing the denominators. Collect like terms.
- Equate the resulting coefficients of the powers of x and solve the resulting system of linear equations.