Quiz 7 solutions

1. Set up the decomposition for the following rational functions as a sum of elementary rational functions as described in Key Idea 6.5.1 (see the original quiz). Your answer should contain several unknowns.

   (a) $\dfrac{3x - 7}{x^2 - 4}$

   Solution. The decomposition is

   $$\frac{3x - 7}{x^2 - 4} = \frac{3x - 7}{(x + 2)(x - 2)}$$
   $$= \boxed{\frac{A}{x + 2} + \frac{B}{x - 2}},$$

   where A, B are the unknowns.   □

   (b) $\dfrac{x^2 - 2x + 3}{x^3 + 5x}$

   Solution. The decomposition is

   $$\frac{x^2 - 2x + 3}{x^3 + 5x} = \frac{x^2 - 2x + 3}{x(x^2 + 5)}$$
   $$= \boxed{\frac{A}{x} + \frac{Bx + C}{x - 2}},$$

   where A, B, C are the unknowns.   □

   (c) $\dfrac{x^3 - x + 1}{(x^2 + 2)^2}$

   Solution. The decomposition is

   $$\frac{x^3 - x + 1}{(x^2 + 2)^2} = \frac{x^3 - x + 1}{(x^2 + 2)(x^2 + 2)}$$
   $$= \boxed{\frac{Ax + B}{x^2 + 2} + \frac{Cx + D}{(x^2 + 2)^2}},$$

   where A, B, C, D are the unknowns.   □

2. Solve for all of the unknowns in the equation

   $$\frac{3x^2 + 7x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}.$$

   Solution. We can multiply both sides of the equation

   $$\frac{3x^2 + 7x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}.$$

   by $x(x^2 + 3)$ to write

   $$3x^2 + 7x + 6 = (x^2 + 3)A + x(Bx + C)$$
   $$= Ax^2 + 3A + Bx^2 + Cx$$
   $$= (A + B)x^2 + Cx + 3A.$$

   Next, we can equate the coefficients to obtain the linear system of equations

   $$A + B = 3,$$
   $$C = 7,$$
   $$3A = 6.$$

   This linear system of equations has the solutions

   $$A = \boxed{2},$$
   $$B = \boxed{1},$$
   $$C = \boxed{7},$$

   as desured,   □