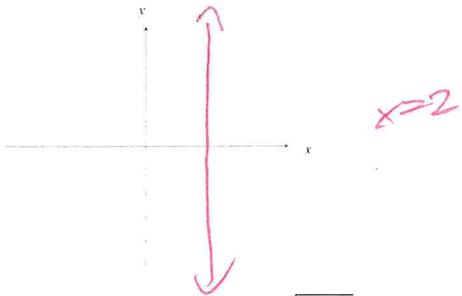
Name: Solution

Quiz 1

MATH 022, Section 008
University of California, Riverside
October 8, 2018

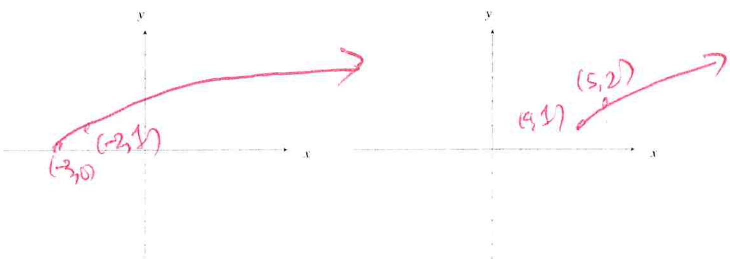
This quiz is worth 15 points. You have 30 minutes to complete the quiz. If you need more space, continue your work on the back side of the page and write "see back" next to your work on the front side of the page accordingly.

3 pts) 1. Find the equation of a line that passes through the points (2, 3) and (2, −2). Graph the line in the provided set of axes below.



$x = 2$

3 pts) 2. Sketch the graphs of $y = \sqrt{x+3}$ and $y = \sqrt{x-4} + 1$ in the provided sets of axes below.



3 pts) 3. Evaluate $\lim_{h\to 0} \frac{\sqrt{x+1}-2}{x-3}$. (handwritten correction: $x \to 3$)

** Everyone got 3 points automatically ** ~~because of the typo~~ on the question.

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$

$$= \lim_{x\to 3} \frac{(\sqrt{x+1})^2 - 2^2}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2}$$

$$= \frac{1}{\sqrt{3+1}+2} = \boxed{\frac{1}{4}}$$

3 pts) 4. Describe the interval(s) on which the function $f(x) = \frac{x-5}{x^2-9x+20}$ is continuous. Note any point(s) of discontinuity, and determine the type of each discontinuity (removable, jump, or infinite).

$$f(x) = \frac{x-5}{x^2-9x+20} = \frac{x-5}{(x-5)(x+4)}$$

f is continuous on $(-\infty, 4) \cup (4, 5) \cup (5, \infty)$

Since the "$x-5$"'s were cancelled out, $x=5$ is a hole, and therefore $x=5$ is a removable discontinuity.

Since we cannot cancel out "$x+4$" in the denominator, $x=4$ is an asymptote, and therefore $x=4$ is an infinite discontinuity.

3 pts) 5. Use the limit definition to compute the derivative of $f(x) = \frac{1}{x+2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+2}\cdot\frac{x+2}{x+2} - \frac{1}{x+2}\cdot\frac{x+h+2}{x+h+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+2) - (x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} -\frac{1}{(x+h+2)(x+2)}$$

$$= \boxed{-\frac{1}{(x+2)^2}}$$