Name: Solution

Quiz 2
MATH 022, Section 008
University of California, Riverside
October 16, 2018

This quiz is worth 15 points. You have 30 minutes to complete the quiz. If you need more space, continue your work on the back side of the page and write "see back" next to your work on the front side of the page accordingly.

(3 pts) 1. Find an equation of the tangent line to the graph of the function $y = -2x^4 + 5x^2 - 3$ at the point $(1, 0)$.

$y = -2x^4 + 5x^2 - 3$

$\Rightarrow \frac{dy}{dx} = -8x^3 + 10x$

So at $x=1$, the slope is

$\left.\frac{dy}{dx}\right|_{x=1} = -8(1)^3 + 10(1)$

$= -8 + 10$

$= 2$

$\longrightarrow$ Therefore,

$y - 0 = 2(x-1)$

$\Rightarrow \boxed{y = 2x - 2}$

(3 pts) 2. The cost C in dollars of producing x units is given by the function $C(x) = 100(9 + 3\sqrt{x})$. Find the marginal cost of producing 4 units.

$C(x) = 100(9 + 3\sqrt{x})$

$= 100(9 + 3x^{\frac{1}{2}})$

$\Rightarrow C'(x) = 100\frac{d}{dx}(9 + 3x^{\frac{1}{2}})$

$= 100(0 + 3 \cdot \frac{1}{2}x^{-\frac{1}{2}})$

$= 100(0 + \frac{3}{2\sqrt{x}})$

$= 100 \cdot \frac{3}{2\sqrt{x}}$

$= \frac{150}{\sqrt{x}}$

$\Rightarrow C'(4) = \frac{150}{\sqrt{4}}$

$= \frac{150}{2}$

$= \boxed{75}$

(3 pts) 3. Find the derivative of $f(x) = \frac{(x-2)(3x+1)}{4x+2}$. ← expanding this before using quotient rule works also (bypasses having to use product rule)

quotient rule

$f'(x) = \frac{(4x+2)\frac{d}{dx}\left((x-2)(3x+1)\right) - \left((x-2)(3x+1)\right)\frac{d}{dx}(4x+2)}{(4x+2)^2}$

product rule

$= \frac{(4x+2)\left(\frac{d}{dx}(x-2)\cdot(3x+1) + (x-2)\frac{d}{dx}(3x+1)\right) - \left((x-2)(3x+1)\right)\cdot 4}{(4x+2)^2}$

$= \frac{(4x+2)(1\cdot(3x+1) + (x-2)\cdot 3) - 4(x-2)(3x+1)}{(4x+2)^2} = \frac{(4x+2)(3x+1+3x-6) - 4(x-2)(3x+1)}{(4x+2)^2}$

$= \boxed{\frac{(4x+2)(6x-5) - 4(x-2)(3x+1)}{(4x+2)^2}}$

(3 pts) 4. Find the derivative of $f(x) = x(3x-9)^3$.

$f'(x) = \frac{d}{dx}\left(x(3x-9)^3\right)$

product rule

$= \frac{d}{dx}(x)\cdot(3x-9)^3 + x\cdot\frac{d}{dx}(3x-9)^3$

chain rule

$= 1\cdot(3x-9)^3 + x\cdot\left(3(3x-9)^2\cdot\frac{d}{dx}(3x-9)\right)$

$= (3x-9)^3 + x\cdot(3(3x-9)^2\cdot 3)$

$= \boxed{(3x-9)^3 + 9x(3x-9)^2}$

(3 pts) 5. Given $f(x) = (x^3 - 2x)^3$, evaluate $f''(1)$.

$\Rightarrow f'(x) = \frac{d}{dx}(x^3-2x)^3$

chain rule

$= 3(x^3-2x)^2\frac{d}{dx}(x^3-2x)$

$= 3(x^3-2x)^2\cdot(3x^2-2)$

$\Rightarrow f''(x) = \frac{d}{dx}\left(3(x^3-2x)^2(3x^2-2)\right)$

product rule

$= 3\left(\left(\frac{d}{dx}(x^3-2x)^2\right)(3x^2-2) + (x^3-2x)^2\cdot\frac{d}{dx}(3x^2-2)\right)$

chain rule

$= 3\left(2(x^3-2x)\cdot\frac{d}{dx}(x^3-2x)\right)(3x^2-2) + (x^3-2x)^2\cdot 6x)$

$= 3\left(2(x^3-2x)\cdot(3x^2-2)\right)(3x^2-2) + 6x(x^3-2x)^2) = 6(x^3-2x)(3x^2-2)^2 + 18x(x^3-2x)^2$

$\longrightarrow f''(1) = 6((1)^3-2(1))(3(1)^2-2)^2 + 18(1)((1)^3-2(1))^2$

$= 6(1-2)(3-2)^2 + 18(1-2)^2$

$= 6\cdot(-1)\cdot(1)^2 + 18\cdot 1\cdot(-1)^2$

$= -6 + 18$

$= \boxed{12}$