Name: Solution

Quiz 6
MATH 022, Section 008
University of California, Riverside
December 4, 2018

This quiz is worth 15 points. You have 30 minutes to complete the quiz. If you need more space, continue your work on the back side of the page and write "see back" next to your work on the front side of the page accordingly.

(3 pts) 1. Evaluate $\int x\ln(x+1)\,dx$.

$u = \ln(x+1)$ $\quad dv = x\,dx$

$du = \frac{1}{x+1}\,dx$ $\quad v = \frac{x^2}{2}$

$\int u\,dv = uv - \int v\,du$

$\int x\ln(x+1)\,dx = (\ln(x+1))\left(\frac{x^2}{2}\right) - \int\left(\frac{x^2}{2}\right)\left(\frac{1}{x+1}dx\right)$

$= \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\int\frac{x^2}{x+1}dx$

$= \boxed{\frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\left(\frac{(x+1)^2}{2} - 2(x+1) + \ln|x+1| + C\right)}$

Let $u = x+1$, Then $du = dx$ and $x = u-1$.

$\int\frac{x^2}{x+1}dx = \int\frac{(u-1)^2}{u}du$

$= \int\frac{u^2-2u+1}{u}du$

$= \int u - 2 + \frac{1}{u}\,du$

$= \frac{u^2}{2} - 2u + \ln|u| + C$

$= \frac{(x+1)^2}{2} - 2(x+1) + \ln|x+1| + C$

(3 pts) 2. Evaluate $\int_5^\infty \frac{x}{\sqrt{x^2-16}}\,dx$.

Let $u = x^2 - 16$.
Then $du = 2x\,dx$.

$\int_5^\infty\frac{x}{\sqrt{x^2-16}}dx = \frac{1}{2}\int_5^\infty\frac{2x}{\sqrt{x^2-16}}dx$

$= \frac{1}{2}\int_{u(5)}^{u(\infty)}\frac{1}{\sqrt{u}}du$

$= \frac{1}{2}\int_{u(5)}^{u(\infty)}u^{-\frac{1}{2}}du$

$= \frac{1}{2}\left(\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right)\Big|_{u(5)}^{u(\infty)}$

$= \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\Big|_{u(5)}^{u(\infty)}$

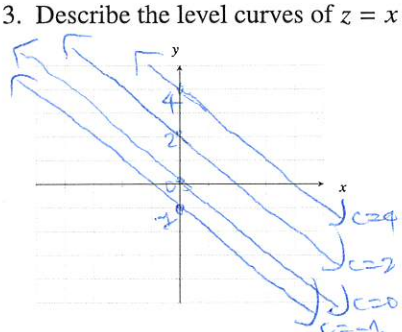
$= \sqrt{u}\Big|_{u(5)}^{u(\infty)}$

$= \sqrt{x^2-16}\Big|_5^\infty$

$= \lim_{a\to\infty}\sqrt{a^2-16} - \sqrt{(5)^2-16}$

$= \infty - 3 = \boxed{\infty}$ $\boxed{\text{diverges}}$

(3 pts) 3. Describe the level curves of $z = x + y$ with $c = -1, 0, 2, 4$. Sketch a contour map of the surface using level curves for these c-values.



$-1 = x+y$ $\quad y = -x-1$
$0 = x+y$ $\quad \Longrightarrow \quad y = -x$
$2 = x+y$ $\quad y = -x+2$
$4 = x+y$ $\quad y = -x+4$

(3 pts) 4. Find the first partial derivatives of $f(x, y) = e^{3xy}$ and evaluate the partial derivatives at $(0, 4)$.

$f_x(x,y) = \frac{\partial}{\partial x}(e^{3xy})$
$= e^{3xy}\cdot\frac{\partial}{\partial x}(3xy)$
$= e^{3xy}\cdot 3y$
$= 3ye^{3xy}$

$f_x(0,4) = 3(4)e^{3(0)(4)}$
$= 12\cdot 1$
$= \boxed{12}$

$f_y(x,y) = \frac{\partial}{\partial y}(e^{3xy})$
$= e^{3xy}\cdot\frac{\partial}{\partial y}(3xy)$
$= e^{3xy}\cdot 3x$
$= 3xe^{3xy}$

$f_y(0,4) = 3(0)e^{3(0)(4)}$
$= 0\cdot 1$
$= \boxed{0}$

(3 pts) 5. Find the first partial derivatives of $f(x, y) = 200x^{0.7}y^{0.3}$ and evaluate the partial derivatives at $(1000, 500)$.

$f_x(x,y) = \frac{\partial}{\partial x}(200x^{0.7}y^{0.3})$
$= 200(0.7x^{0.7-1})y^{0.3}$
$= 200\cdot 0.7x^{-0.3}y^{0.3}$

$f_x(1000,500) = \boxed{200\cdot 0.7(1000)^{-0.3}(500)^{0.3}}$

$f_y(x,y) = \frac{\partial}{\partial y}(200x^{0.7}y^{0.3})$
$= 200x^{0.7}(0.3y^{0.3-1})$
$= 200\cdot 0.3x^{0.7}y^{-0.7}$

$f_y(1000,500) = \boxed{200\cdot 0.3(1000)^{0.7}(500)^{-0.7}}$