Discussion 7/10/19

Proof by contradiction:

Example 1: Prove that, if n^2 is even, then n is even.

Proof: Suppose by contradiction n is odd, then $\exists$ int k where $n = 2k+1$

$$n^2 = 4k^2 + 4k + 1 = 2(2k^2+2k)+1$$

Since $2k^2+2k$ is also an int, so n^2 is odd. But this contradicts n^2 is even. So this completes our proof by contradiction, & we conclude that n is even.

Ex 2: Prove that $\sqrt{2}$ is irrational.

Proof: Assume by contradiction $\sqrt{2}$ is rational. Then there $\exists$ ints a, b with no common factors with $b \neq 0$ such that $\frac{a}{b} = \sqrt{2}$

From $\sqrt{2} = \frac{a}{b}$ we see:

$$2 = a^2/b^2$$

$$a^2 = 2b^2$$

Since b^2 is an int, it follows that a^2 is even

If a^2 is even, then a is even

Suppose by contradiction that a is odd

Then $\exists$ int k, where $a = 2k+1$

$$a^2 = 2(2k^2+2k)+1$$

Since $2k^2+2k$ is an int, a^2 is odd.

But this contradicts a^2 is even.

So a must be even.

Since a is even,

$\exists$ int l, where $a = 2l$

$$(2l)^2 = 2b^2$$

$$4l^2 = 2b^2$$

$$2l^2 = b^2 \quad b^2 \text{ is even}$$

Since b^2 is even, b is even. So a & b are both even.

But this means a & b have 2 as a common factor.
This contradicts our assumption that a & b
have no common factors (greater than 1)
So we proved $\sqrt{2}$ is irrational.