Discussion July. 10. 2019

Proof by contradiction

Example 1: Prove that if n^2 is even, then n is even.

Proof: Suppose by contradiction then n is odd.

Then, there exists an integer k, such that

$n = 2k+1$

So we have $n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

Since $(2k^2+2k)$ is also an integer, we conclude that n^2 is odd.

But this contradicts our assaption in the premises that n^2 is even.

$\therefore$ this completes our proof by contradiction, and we conclude that n is even.

Example 2: Prove that $\sqrt{2}$ is irrational.

Proof: Assume by contradiction that $\sqrt{2}$ is rational.

Then there exist integers a, b with no common factors and $b \neq 0$, s.t.

$\sqrt{2} = \frac{a}{b}$

From $\sqrt{2} = \frac{a}{b} \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$

Since b is an integer, it followes that a^2 is even

We claim that if a^2 is even, then a is even.

To prove our claim, suppose contradiction that a is odd.

Then there exists an integer k such that

$$a=2k+1$$

So we have

$$a^2=(2k+1)^2$$
$$=2(2k^2+2k)+1$$

Since $2k^2+2k$ is also an integer, it follows that a^2 is odd. But this contradicts our assumption that a^2 is even

$\therefore$ a is even, satisfying our claim

Now we return to the equation

$$a^2=2b^2$$

$\because$ a is even, there exist an integer l, s.t.

$$a=2k$$

So we obtain

$$(2l)^2=(2b)^2$$
$$4l^2=2b^2$$
$$b^2=2l^2$$

So b^2 is even. by the claim we made earlier we have that b is even.

$\therefore$ a, b are both even.

In other words, 2 is a common factor of a and b.

But this contradicts our assuption earlier in this proof that a and b have no common factors (greater than 1)

So we conclude that $\sqrt{2}$ is irrational.