7/10/2019 Disscussion

Proof by Contradiction

Example 1: Prove that, if n^2 is even, then n is even.

proof: Suppose by contradiction that n is odd then there exists an integer k such that $n = 2k+1$

So we have

$$n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$$

Since $2k^2+2k$ is also an integer, we conclude that n^2 is odd. But this contradicts our assumption in the premises that n^2 is even. So this completes our proof by contradiction, and we conclude that n is even. ☐

Example 2: Prove that $\sqrt{2}$ is irrational.

proof: Assume by contradiction that $\sqrt{2}$ is rational. Then there exist integers a, b — with no common factors greater than 1 and $b \neq 0$ — such that

$$\sqrt{2} = \frac{a}{b}.$$

From $\sqrt{2} = \frac{a}{b}$, we obtain $2 = \frac{a^2}{b^2}$

or equivalently, $a^2 = 2b^2$

Since b^2 is an integer, it follows that a^2 is even.

$\boxed{\text{We claim that: if } a^2 \text{ is even, then } a \text{ is even}}$.

To prove our claim, suppose by contradiction that a is odd.

Then there exists an integer k such that →

$$a = 2k+1$$

So we have

$$a^2 = (2k+1)^2$$
$$= 4k^2 + 4k + 1$$
$$= 2(2k^2 + 2k) + 1$$

Since $2k^2 + 2k$ is also an integer, it follows that a^2 is odd.

But this contradicts our assumption that a^2 is even.

Therefore, a is even, satisfying our claim.

By $\boxed{\text{the claim}}$,

we have that a is even.

Now we return to the equation

$$a^2 = 2b^2$$

Since a is even, there exists an integer l such that

$$a = 2l.$$

So we obtain

$$(2l)^2 = 2b^2$$
$$4l^2 = 2b^2$$
$$2l^2 = b^2$$
$$(b^2 = 2l^2)$$

So b^2 is even.

Since b^2 is even by $\boxed{\text{the claim}}$ we made earlier we have that b is even.

Therefore, a and b are both even. In other words, 2 is a common factor of a and b. But this contradicts our assumption earlier in the proof that a and b have no common factors (greater than 1)

So we conclude that $\sqrt{2}$ is irrational

$\sqrt{2} = \frac{a}{b}$. a, b are integers such that

- $b \neq 0$
- a and b have no common factor greater than 1