Discussion 7/10/19 week3

Proof by contradiction

Example 1: Prove that, if n^2 is even, then n is even.

Proof: Suppose by contradiction that n is odd. Then there exists an integer k such that $n = 2k+1$. So we have

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Since $2k^2 + 2k$ is also an integer, we conclude that n^2 is odd. But this contradicts out assumption in the premises that n^2 is even. So this completes our proof by contradiction, & we conclude that n is even.

Example 2: Prove that the $\sqrt{2}$ is irrational.

Proof: Assume by contradiction that $\sqrt{2}$ is rational. Then there exist integers a, b – w/no common factors $b \neq 0$ – such that $\sqrt{2} = \frac{a}{b}$

From $\sqrt{2} = \frac{a}{b}$, we obtain $2 = \frac{a^2}{b^2}$ or equivalently,

$a^2 = 2b^2$. Since b^2 is an integer, it follows that a^2 is even.

We claim that: if a^2 is even, then a is even.

To prove our claim, suppose by contradiction that a is odd. Then there exists an integer k such that $a = 2k+1$ so we have

$$a^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Since $2k^2 + 2k$ is also an integer, it follows that a^2 is odd. But this contradicts our assumption that a^2 is even. Therefore, a is even, satisfying our claim.

Now we return to the eqth

$$a^2 = 2b^2$$

Since a is even there exists an integer l. such that

$$a = 2l$$

So we obtain

$$(2l)^2 = 2b^2$$
$$4l^2 = 2b^2$$
$$2l^2 = b^2$$
$$(b^2 = 2l^2) \text{ so } b^2 \text{ is even.}$$

Since b^2 is even, by the claim we made earlier we have that b is even.

Therefore, a & b are both even. In other words, 2 is a common factor a & b. But this contradicts our assumption earlier in this proof that a & b have no common factors (greater than 1)

So we conclude that $\sqrt{2}$ is irrational.

$\sqrt{2} = \frac{a}{b}$

a, b are integers such that

- $b \neq 0$
- a & b have no common factor greater than 1.